

EXPRESS MAIL NO.: TB304043190US

Mailing Date: July 21, 1999

Attorney Docket No. 11686US02

1c530 U.S. PTO
09/362635
07/21/99

McAndrews, Held & Malloy, Ltd.
34th Floor
500 West Madison Street
Chicago, Illinois 60661
(312) 707-8889

July 21, 1999

Assistant Commissioner for Patents
BOX PATENT APPLICATION
Washington, D.C. 20231

Sir:

Transmitted herewith for filing is a continuation/division patent application under 37 CFR § 1.53(b) of a parent application, U.S. Serial No.08/641,061, which is NOT to be abandoned of Inventor(s): Rajiv Ramaswami and Galen H. Sasaki for **"MULTICHANNEL RING AND STAR NETWORKS WITH LIMITED CHANNEL CONVERSION."**

This application contains no new matter, with reference to the parent application.

1. Enclosed are:
 - A. A Declaration and Power of Attorney signed by each inventor which is a photocopy of a Declaration and Power of Attorney signed by each inventor and previously filed in U.S. Serial No. 08/641,061;
 - B. A Power of Attorney signed by an officer of Tellabs, Inc., the titleholder of record;
 - C. 9 sheet(s) of drawing(s) (informal);
 - D. A first assignment of rights which is a photocopy of a signed assignment of U.S. Serial No. 08/641,061 from the inventors to International Business Machines Corporation recorded at Reel/Frame 8087/0158 on July 22, 1996;

OIPE/JCWS

JUL 30 1999

RECEIVED

- E. A second assignment of rights which is a photocopy of a second Assignment of U.S. Serial No. 08/641,061 from International Business Machines Corporation to Tellabs, Inc., recorded at Reel/Frame 8351/0156; and
- F. A Preliminary Amendment canceling claims 1-10.
- G. A copy of the originally filed application retyped using double spacing as

requested by the Examiner in the parent application.

2. Benefit of the filing date of each foreign priority document or U.S. provisional patent application listed below (if any) is claimed under 35 U.S.C. Section 119:

<u>Docket No.</u>	<u>Country</u>	<u>Appln. Serial No.</u>	<u>Filing Date</u>
-------------------	----------------	------------------------------	--------------------

3. The filing fee has been calculated as shown below, based on the assignee's status as a large entity:

	Claims Present		Extra Claims	Rate Per Extra Claim	Total Fee
Total Claims	13	- 20 =	~	x \$22.00	.00
Indep Claims	8	- 3 =	5	x \$78.00	\$390.00
Basic Fee					\$760.00
Multiple dependent claim presented (\$270 if any present)					.00
Total filing fee					\$1,150.00

4. The following arrangements have been made to pay the filing fee:

- A. A check in the amount of \$1,150.00 to cover the filing fee is enclosed
- B. The Commissioner is hereby authorized to charge any additional filing fees which may be required under 37 C.F.R. 1.16 for this application and for any amendment thereto, or credit any overpayment to Account No. 13-0017.

5. The Commissioner is hereby authorized to charge payment of the following fees during the pendency of this application or credit any overpayment to Deposit Account No. 13-0017.

- A. Any patent application processing fees under 37 C.F.R. 1.17.

6. Please address all telephone calls to:

Ronald E. Larson at telephone No. (312) 707-8889.

7. Please address all correspondence to:

Ronald E. Larson
McAndrews, Held & Malloy, Ltd.
34th Floor
500 West Madison Street
Chicago, Illinois 60661



Ronald E. Larson
Reg. No. 24,478

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

Case No. 11686US02

PATENT

In the Application of:

Rajiv Ramaswami et al.

Serial No.:

Filed:

For: MULTICHANNEL RING AND STAR
NETWORKS WITH LIMITED CHANNEL
CONVERSION

Examiner:

Group Art Unit:

CERTIFICATE OF MAILING

I hereby certify that this
correspondence and all correspondence
referenced herein are being deposited with
the United States Postal Service as first
class mail in an envelope addressed to the
Asst. Commissioner for Patents, United
States Patent and Trademark Office,
Washington DC 20231 on July 21, 1999.

By: Ronald E. Larson
Ronald E. Larson
Reg. No. 24,478

PRELIMINARY AMENDMENT

Commissioner of Patents and Trademarks
Washington, DC 20231

Dear Sir:

Kindly amend the above-identified divisional application filed under Rule 53(b) as
follows:

In the specification:

After the title kindly insert:

-- Cross-Reference To Related Applications

This is a division of U.S. Application Serial No. 08/641,061, entitled "Multichannel
Ring And Star Networks With Limited Channel Conversion," filed April 29, 1996 in the
names of Ramaswami et al.

Statement Regarding Federally Sponsored Research Or Development

The invention herein was made with Government support under Contract MDA972-95-C-0001 awarded by the Advanced Research Projects Agency (ARPA).

The Government has certain rights in this invention.--

In the claims:

Kindly cancel claims 1-10. Kindly add new claims 17-29 as follows:

17. In a star communication network comprising a hub node and comprising a plurality of links coupled to said hub node for carrying data in W channels along routes, a method of configuring said network comprising the steps of:

limiting said W channels to an even number;

dividing said W channels into a first group and a second group in each of said links;

connecting each channel of said first group of one of said links to one channel of said second group of each of said links other than said one link; and

assigning no more than W channels to the transmission of data along any of said links, whereby the efficiency of the configuring is improved.

18. A method, as claimed in claim 17, and further comprising the step of assigning said routes to said channels which traverse at most two of said links.

19. A method, as claimed in claim 17, wherein said step of connecting comprises the step of connecting each channel $i = 0, 1, \dots, W/2 - 1$ of a first one of said links through said hub node to channel $w(i)$ on each of said links other than said first link where $w(i) = i + W/2$.

20. A star communication network comprising in combination:

a hub node; and

links coupled to said hub node for carrying data along routes in W channels divided into a first group and a second group where W is even, said hub node comprising switches connecting each channel of said first group of one of said links to one channel of said second group of each of said links other than said one link.

21. A network, as claimed in claim 20, wherein each said link comprises no more than W channels.

22. A network, as claimed in claim 20, wherein said routes traverse at most two of said links.

23. Apparatus, as claimed in claim 20, wherein each channel $i = 0, 1, \dots, W/2 - 1$ of said one link is connected through said hub node to channel $w(i)$ on all of said links other than said one link where $w(i) = i + W/2$.

24. In a star communication network comprising a hub node and comprising links coupled to said hub node for carrying data in W channels, a method of configuring said network comprising the steps of:

assigning no more than W channels to the transmission of data along any of said links; and

connecting each channel of a first one of said links to no more than two channels of a second one of said links through said hub node, whereby the efficiency of the configuring is improved.

25. A star communication network comprising in combination:

a hub node; and

links coupled to said hub node for carrying data along routes in no more than W channels, said hub node comprising switches connecting each channel of a first one of said links to no more than two channels of a second one of said links through said hub node.

26. In a star communication network comprising a hub node and comprising links coupled to said hub node for carrying data in W channels, a method of configuring said network comprising the steps of:

assigning no more than W channels to the transmission of data along any of said links; and

connecting each channel of a first one of said links to no more than a second channel of a second one of said links through said hub node, where the second channel is different from the first channel of the second one of said links.

27. A star communication network comprising in combination:

a hub node; and

links coupled to said hub node for carrying data along routes in no more than W channels, said hub node comprising switches connecting each channel of a first one of said links to no more than a second channel of a second one of said links through said hub node, where the second channel is different from the first channel of the second one of said links.

28. A method of proposing a star network comprising:
proposing a network comprising a hub node;
proposing links coupled to said hub node for carrying data along routes in W channels divided into a first group and a second group where W is even; and
proposing that said hub node comprise switches connecting each channel of said first group of one of said links to one channel of said second group of each of said links other than said one link.

29. A method of proposing a star communication network comprising:
proposing a hub node;
proposing links coupled to said hub node for carrying data along routes in no more than W channels; and
proposing that said hub node comprise switches connecting each channel of a first one of said links to no more than two channels of a second one of said links through said hub node.

REMARKS

This is a divisional application which presents the subject matter of claims 11-16 and 29-39 which were subject to a restriction requirement in the Office Action mailed Feb. 1, 1999 in the parent application. Claims 11-16 of this application are the same as original claims 11-16 of parent application. Claims 17-27 are the same as claims 29-39 added by amendment to the parent application. New claims 28-29, like allowed claim 43 of the parent application, are directed to a method of proposing a network. Such claims are allowable under the holding in *State Street Bank & Trust Co. vs. Signature Financial Group, Inc.*, 149 F.3d 1368 (Fed. Cir. 1998). In that case, the court laid to rest

the business method exception to statutory subject matter. In particular it referenced MPEP section 706.03(a) (1994) which advises Examiners that claims should not be categorized as methods of doing business. Instead, such claims should be treated as any other process claims.

Enclosed with this Preliminary Amendment is a copy of the originally filed application retyped using double spacing as requested by the Examiner in the parent application.

It is believed that all claims now pending in this application are in condition for allowance, and such action is respectfully solicited.

Date: July 21, 1999

Respectfully submitted,



Ronald E. Larson
Reg. No. 24,478
Attorney for Applicant

McAndrews, Held & Malloy, Ltd.
500 W. Madison, 34th Floor
Chicago, IL 60661
312.707.8889

MULTICHANNEL RING AND STAR NETWORKS WITH LIMITED CHANNEL CONVERSION

Technical Field

The present invention relates generally to multichannel, specifically optical multichannel, networks with ring and star topologies.

Prior Art

A multichannel link comprises a number of channels, say W , between two sites. These channels may be transmitted separately (for example over parallel wires or fiber cables) or multiplexed on to one of a small number of wires or fibers using time or channel division multiplexing. Usually these links are realized in the form of line cards, one for each channel at each end of the link. A line card is a device that provides an interface between the I/O for the channel and the transmission medium. The set of line cards associated with each end of a link along with any associated multiplexing/demultiplexing unit is called a multiplexor.

One example is the IBM optical multiplexer system [1]. This system multiplexes up to ten full-duplex channels on to a single transmission link.

Multiplexors can be connected in a ring or star network configuration across multiple sites (herein called nodes). Nodes may be configured to allow pairs of channels to be connected to one another. This may be accomplished by some kind of switching at the node. For example, consider a network realized by line cards. In addition, consider two channels from different links, but where the links are incident to a common node. Each of these channels has a line card at the node. Suppose the line cards are connected. Then the channels may be connected to each other since the signal from one channel may be transferred to the other channel by going through the line cards and the connection between the line cards. If a pair of channels may be connected to one another, as for example, through a switching network, then we refer to them as being *attached*.

A node is said to be *configured* if pairs of its incident channels are attached. The network is said to be *configured* if each of its nodes is configured. For a network configuration, a node is said to have *channel degree* k if for each pair of its incident links, the channels of the links have the following property: each channel in one link is attached to k channels of the other link. A node has *full channel conversion* if its channel degree is W . A node is said to have *fixed channel conversion* if its channel degree is one. Suppose at each link in the network, the channels are numbered $\{0, 1, \dots, W - 1\}$. Then a node is said to have *no channel conversion* if its channel degree is such that channels with the same number are attached.

A network is configured so that end-to-end communication connections between pairs of nodes may be established in the network. An end-to-end communication connection is

specified by a path in the network, and it is realized by a set of channels, one from each link along the path so that channels that are incident to a common node are attached through the node. This realization allows a signal that is sent from one end of the path to be received at the other end by being transported along the attached channels. The path corresponding to an end-to-end communication connection will be referred to as a *route*, and a set of channels that realizes the end-to-end communication connection will be referred to as a *channel assignment* for the route.

Note that it is straightforward to realize a set of end-to-end communication connections in a network configured so that each node has full channel conversion. It is more cost effective to have nodes configured so that some or all nodes have channel degree less than W , i.e., allow only limited switching capability at the nodes. However, in general, networks configured to have less than full channel conversion at each node may require more channels to realize the same end-to-end communication connections than if they were configured to have full channel conversion at each node.

A *request* is a set of routes and corresponds to a set of end-to-end communication connections. The *load* of a request is the value $\max_{e \in E} \lambda_e$, where λ_e denotes the number of routes using link e and E denotes the set of links in the network. For a network configuration, a *channel assignment* for a request is a collection of assignments for routes, one per route of the request, such that each channel is assigned to at most one route of the request, i.e., no two routes will share a channel. Note that a channel assignment for a request realizes all of the end-to-end communication connections corresponding to the request.

Prior art focuses on networks with either no channel conversion or networks with full

channel conversion. For the case where all nodes have full channel conversion, (i.e., $k = W$), a sufficient (and necessary) condition for feasibility is $W \geq \lambda_{\max}$, where λ_{\max} is the load for the request. For the case when all nodes have no channel conversion (hence at each node, $k = 1$), [2] gives a method that performs a channel assignment using $W \geq 2\lambda_{\max}$ on a ring network and $W \geq \frac{3}{2}\lambda_{\max}$ for a star network.

Prior art also proposes several heuristic channel assignment schemes for networks without channel conversion that may not be efficient in terms of using a small number of channels to perform the channel assignment. For example, see [3, 4, 5, 6, 7, 8]. For the case of limited channel conversion, [9, 10] propose some network configurations and some heuristic channel assignment schemes for these configurations that again may not be efficient in terms of using a small number of channels to perform the channel assignment. Prior art does not propose configuration methods and efficient channel assignment techniques for networks with limited channel conversion.

Summary of the Invention

The invention proposes configurations of ring and star networks with limited channel conversion to efficiently support connections. In addition, algorithms are provided to efficiently assign channels to connections.

More specifically, it is an object of this invention to provide a cost effective network by using nodes with limited switching capability.

It is also an object of this invention to efficiently assign channels to links of a network with nodes having limited switching capabilities so as to maximize network resources.

More generally, it is the overall object of this invention to configure a network and assign channels to the network in a cost effective manner.

The invention achieves the following results:

In a *ring* network with N nodes, the invention proposes a network configuration and for this configuration, proposes a channel assignment method for any request with load λ_{\max} that uses

- λ_{\max} channels with channel degree at most $k = 2$ at each node provided $N \geq 2 \log_2 \lambda_{\max} - 1$ and W is a power of two.
- λ_{\max} channels with channel degree at most $\Delta + 1$ at each node, where $\Delta > 1$, provided $N \geq \log_{\Delta} \lambda_{\max}$.

In a *star* network, the invention proposes a configuration and for this configuration, proposes a channel assignment method for any request with load λ_{\max} that uses λ_{\max} channels with *fixed conversion*.

In a network with an arbitrary topology the invention proposes a configuration and for this configuration, proposes a channel assignment method for any request with load λ_{\max} where no connection is more than 2 hops, that uses λ_{\max} channels with *fixed conversion*.

Accordingly, this invention provides for a method of configuring nodes in a ring communications network wherein one of the nodes of ring is designated as a primary node, which is configured to have full channel conversion. That is, any two channels between its two incident links can be connected to each other. The other nodes of the network are configured to have no channel conversion. That is, any channel c on one of the incident links of a node is connected to the same channel c on the other incident link of the node.

This invention also provides a method of assigning channels in a ring communications network which is configured as described in the previous paragraph. With the assignment scheme of the invention the paths used for end-to-end communication connections are divided into cut paths and uncut paths, where a cut path is a path that passes through the primary node, while an uncut path does not pass through the primary node. Each cut path p_i is divided into two paths a_i and b_i by splitting path p_i at the primary node, where the primary node becomes an end node for paths a_i and b_i . The paths a_i and b_i are referred to as residual paths. Then, each link along each uncut path is assigned a single channel, and each link along a residual path is assigned the same channel. Thus, a cut path can use two different channels corresponding to its two residual paths.

This invention also comprises a network which is configured as above.

This invention also provides a method of configuring nodes of a ring communications network having multichannel multiplexed links. With this method, the N nodes of the ring are numbered consecutively starting at the primary node and proceeding in one direction around the ring. Also, the link between nodes i and $(i + 1) \bmod N$ is given the number i . Then, each of the nodes is configured such that channel c on link i may be connected to one of $\Delta + 1$ channels on link $(i + 1) \bmod N$, where Δ is greater than or equal to 2 and where one of the channels on link $(i + 1) \bmod N$ is channel $(C + 1) \bmod W$, where the other Δ channels on link $(i + 1) \bmod N$ are channels $(C - k \cdot \Delta^i) \bmod W$ for $k = 0, 1, \dots, \Delta - 1$ and where W is the number of channels in each link. This invention also describes a method of assigning the channels in a ring communications network configured as described in the previous paragraph, and the details of this assignment scheme is described in claim 9 and in

the specification.

This invention also describes a method of configuring the nodes in an arbitrary network having N nodes and E links, where each link is a multichannel multiplexed link having W channels, and where W is an even integer. With this aspect of the invention, the channels are numbered from 0 to $W - 1$, and at each node, for channels $i = 0, 1, \dots, W/2 - 1$, channel i on one link is connected to channel $w(i)$ on all other links incident to that node, where $w(i) = i + W/2$.

A final aspect of the invention is a method of assigning channels to the arbitrary network configured as in the previous paragraph. This assignment scheme is described in the specification and in claim 15.

Brief Description of the Drawings

Fig. 1 shows a configuration of multiplexors in a ring network for the case of full conversion at one node and no conversion at the other nodes.

Fig. 2 shows a simplified diagram of a 4-node ring network and a sample request.

Fig. 3 shows the graph H , representing a Benes permutation network for the case of 4 wavelengths ($W = 4$) along with a set of edge-disjoint paths in H .

Fig. 4 shows a configuration of multiplexors in a ring network corresponding to a Benes network configuration.

Fig. 5 shows the setting of the switches and the channel assignment for the request of Fig. 2 in a ring network with channel degree 2 for the configuration of Fig. 4.

Fig. 6 shows a configuration of multiplexors in a ring network for the case of channel degree

3.

Fig. 7 shows the setting of the switches and the channel assignment for the request of Fig. 2 in a ring network with channel degree 3 for the configuration of Fig. 6.

Fig. 8 shows a configuration of multiplexors in a star network with fixed channel conversion.

Fig. 9 (A) shows a simplified diagram of a star network with 4 end nodes and a sample request of routes. (B) shows how to direct the routes as described in the embodiment of the invention. (C) shows the construction of a bipartite graph and channel assignments for this request.

Fig. 10 shows the setting of the switches and the channel assignment for the request of Fig. 9 for the configuration of Fig. 8.

Description of the Preferred Embodiment

Ring Network

Fig. 1 shows the block diagram of multiplexors 101 connected in a ring network configuration. Each node 102 in the network consists of a pair of multiplexors. Two nodes are connected by a transmission link or medium 103. The figure shows 4 channels on each link. For each channel there is a line card 104 within each multiplexor 101. A line card consists of an I/O port 105, multiple local ports 106 and a line port 107 and a switch (not shown in the figure) that allows any pairs of these ports to be connected together. For the case of the ring network, the number of local ports per line card is at least the channel degree defined earlier. In Fig. 1 node 0 has channel degree 4 while other nodes have channel degree 1. Node

0 is called the *primary* node. The line ports of all the line cards within a multiplexor are connected to a mux/demux unit 108 which combines all the channels on to the transmission link. Within each node the line cards from one multiplexor are hard wired to the line cards in the other multiplexor according to a specific wiring pattern 109 given later. This wiring pattern determines which channels are attached to each other within the node. In node 0 for example, each channel is attached to all the channels. In the other nodes each channel is attached only to other channels with the same channel number.

In the subsequent discussion, we will provide feasibility results for the following network configurations: (i) one node has full channel conversion and the other nodes have no channel conversion, (ii) all nodes have channel degree at most two, (iii) all nodes have channel degree at most $\Delta + 1$, where Δ is an integer greater than one. In the discussion, we will assume, without loss of generality, the following:

- each link has its channels numbered $\{0, 1, \dots, W-1\}$, where W is the number of channels per link;
- nodes are numbered $0, 1, \dots, N-1$ around the ring, where N denotes the number of nodes; and
- for each $i = 0, 1, \dots, N-1$, the link between node i and node $(i+1) \bmod N$ is numbered i .

Configuration with Full Channel Conversion at One Node and No Channel Conversion at Other Nodes

The ring network is configured so that one of its nodes has full channel conversion. This node is referred to as the *primary node*, and without loss of generality, let it be node 0. The other nodes have no channel conversion.

Suppose we are given a request $\{p_1, \dots, p_m\}$, where m is the number of routes in the request. Then the following is a channel assignment for the request. First, refer to routes that pass through node 0 as *cut routes* and the rest of the routes as *uncut*. A set P of paths is generated as follows. Include each uncut route in P . For each cut route p_i , cut (or split) it at node 0 into a pair of paths $\{a_i, b_i\}$ called *residual paths* such that each residual path includes node 0. Without loss of generality, let a_i correspond to the residual path that traverses link $N-1$, and let b_i correspond to the residual path that traverses link 0. Refer to a_i as the *left* residual path, and b_i as the *right* residual path. (For example, if $N = 5$ and p_i is a path with the sequence of nodes $4-5-0-1-2$ then the residual path a_i corresponds to $4-5-0$ and b_i corresponds to $0-1-2$). Include the residual paths in P .

Next, partition the paths in P into W subsets $(P_0, P_1, \dots, P_{W-1})$ such that paths in the same subset do not traverse common links of the ring network. We will refer to the partition $(P_0, P_1, \dots, P_{W-1})$ as a *cut-and-color* partition for the request. One way to find a cut-and-color partition is to assign channel numbers $\{0, \dots, W-1\}$ to the paths in P such that paths with a common link have distinct numbers. This is like *coloring* paths in an *interval graph* [11, Sec.16.5] because no path of P crosses through node 0. Hence, we can use a *greedy algorithm* assignment that requires λ_{\max} numbers [11, Sec.16.5]). [11] is hereby incorporated

by reference. Then for $i = 0, 1, \dots, W - 1$, all paths that have been assigned to channel number i are in subset P_i .

We will now describe the channel assignment for the request. For each uncut route p_i , channel number j is assigned to it where j satisfies $p_i \in P_j$. For each link traversed by p_i , the channel numbered j of that link is assigned to p_i . For each cut route p_i , two channel numbers j_a and j_b are assigned to it, where the channel numbers correspond to the left residual path a_i and right residual path b_i of p_i . In particular, j_a satisfies $a_i \in P_{j_a}$, and j_b satisfies $b_i \in P_{j_b}$. For each link traversed by p_i , a channel is assigned to p_i as follows. If the link is traversed by a_i then the channel numbered j_a is assigned to p_i . Otherwise, the link must be traversed by b_i , and the channel numbered j_b is assigned to p_i .

The desired channel assignment can be realized by setting the switches in the configured network appropriately, as shown in the example below.

Example: Consider the 4-node network of Fig. 1 redrawn in Fig. 2 with $W = 4$ channels and let the request be

$$p_0 = 0 - 1 - 2$$

$$p_1 = 1 - 2 - 3$$

$$p_2 = 2 - 3 - 0 - 1$$

$$p_3 = 2 - 3 - 0$$

$$p_4 = 3 - 0 - 1 - 2$$

and

$$p_5 = 1 - 2 - 3$$

be as shown in the figure. Node 0 is the primary node. Then a cut-and-color partition for

the request is the routes with

$$P_0 = \{p_0, a_2\}$$

$$P_1 = \{b_2, p_1, a_4\}$$

$$P_2 = \{b_4, p_3\}$$

and

$$P_3 = \{p_5\}$$

where $a_2 = 2 - 3 - 0$, $b_2 = 0 - 1$ and $a_4 = 3 - 0$, $b_4 = 0 - 1 - 2$. Here a_i and b_i correspond to the cut routes of p_i . Thus the individual routes would be assigned channels as shown below and in Fig. 2.

Route	Links			
	0	1	2	3
p_0	0	0	-	-
p_1	-	1	1	-
p_2	1	-	0	0
p_3	-	-	2	2
p_4	2	2	-	1
p_5	-	3	3	-

Configuration for Channel Degree 2

Suppose W is a power of two and $N \geq 2 \log_2 W - 1$. There is a configuration with channel degree two at each node with the following property. All requests that have load at most W are feasible.

The configuration attaches pairs of channels to form a *permutation* network. To be more specific, channels are attached according to a new graph H , which has the following properties:

- The set of vertices of H may be organized into $s + 1$ *stages*, numbered $0, 1, \dots, s$, where $s \leq N + 1$, such that there are W vertices $\{u_0, \dots, u_{W-1}\}$ at stage 0 and there are W vertices $\{v_0, \dots, v_{W-1}\}$ at stage s . For the sake of discussion, label the vertices at stage 0 $\{u_0, \dots, u_{W-1}\}$ and the vertices at stage s $\{v_0, \dots, v_{W-1}\}$. We will also refer to those stages $i = 1, 2, \dots, s - 1$ (i.e., those that are not stage 0 or stage s) as the *intermediate* stages.
- The set of edges of H are between consecutive stages of vertices such that there are exactly W edges between stages. To be more specific, for $i = 0, 1, \dots, s - 1$, there are W edges between stage i and stage $i + 1$.
- Each vertex in the stage 0 has exactly one incident edge. Each vertex in stage s has exactly one incident edge.

The graph H has the following additional property. Let any function $f(\cdot)$ on $\{0, \dots, W - 1\}$ be called a *permutation* if $(f(0), \dots, f(W - 1))$ are distinct values of $\{0, \dots, W - 1\}$. For example, if $f(\cdot)$ is a function on $\{0, 1, 2, 3\}$ and $(f(0), f(1), f(2), f(3)) = (1, 3, 0, 2)$ then it

would be a permutation on $\{0, 1, 2, 3\}$. Now H has the property that for any permutation $\pi(\cdot)$ on $\{0, \dots, W-1\}$, there is a collection $(\tau(\cdot), h_0, h_1, \dots, h_{W-1})$, where

- $\tau(\cdot)$ is a permutation on $\{0, \dots, W-1\}$;
- $\{h_0, h_1, \dots, h_{W-1}\}$ is a collection of W paths in H ;
- for each $i = 0, 1, \dots, W-1$, path h_i starts at vertex $u_{\tau(i)}$ in stage 0, traverses stages $1, 2, \dots, s-1$ in succession, and ends at vertex $u_{\tau(\pi(i))}$ in stage s ; and
- the paths $\{h_0, \dots, h_{W-1}\}$ do not have common edges in H , i.e., they are *edge disjoint* in H .

We will refer to the collection $(\tau(\cdot), h_0, \dots, h_{W-1})$ as an *interconnection instance* for $\pi(\cdot)$.

The edges of H are assigned to the channels of the ring network as follows. The W edges of H between the vertices in stages 0 and 1 are assigned to the channels of link 0 such that for $i = 0, 1, \dots, W-1$, the edge incident to u_i of stage 0 is assigned to the channel numbered i . The W edges of H between vertices in stages $s-1$ and s are assigned to the channels of link $(s-1) \bmod N$ such that for $i = 0, 1, \dots, W-1$, the edge incident to v_i of stage s is assigned to the channel numbered i . For $i = 1, \dots, s-2$, the W edges of H between the vertices in stages i and $(i+1) \bmod N$ are assigned to the W channels of link $i \bmod N$ in the ring network. (Note that it is possible for two different stages of edges of H to be assigned to the channels of the same link, e.g., if $s = N+1$ then the edges between stages 0 and 1 and the edges between stages $s-1$ and s will both be assigned to the channels in link 0.) We will use the notation that if e is an edge in H then $\gamma(e)$ is the channel it is assigned to.

The ring network is configured as follows. For $i = 1, 2, \dots, s-1$, channels are attached

through node $i \bmod N$ of the ring network as follows: if e and e' are edges of H such that e is between the stages $i - 1$ and i of vertices, e' is between stages i and $i + 1$ of vertices, and e and e' are incident to a common vertex in stage i then the channels $\gamma(e)$ and $\gamma(e')$ are attached through node i . All other nodes of the ring network are configured so that there is no channel conversion.

A particular topology for H that leads to a network configuration of channel degree two at every node is the Benes interconnection network topology [12]. The Benes topology has $s = 2 \log_2 W$, so that it has $2 \log_2 W + 1$ stages of vertices, where the stage 0 vertices $\{u_0, \dots, u_{W-1}\}$ are the *inputs* of the Benes topology and stage s vertices $\{v_0, \dots, v_{W-1}\}$ are the *outputs*. Fig. 3 shows the graph H for the case $W = 4$. Here, there are 5 stages of vertices, where the stage 0 vertices are $\{u_0, u_1, u_2, u_3\}$, the stage 1 vertices are $\{x_0(1), x_1(1)\}$, the stage 2 vertices are $\{x_0(2), x_1(2)\}$, the stage 3 vertices are $\{x_0(3), x_1(3)\}$, and the stage 4 vertices are $\{v_0, v_1, v_2, v_3\}$. Also note that there are exactly $W = 4$ edges between consecutive stages of vertices.

Notice that in a Benes topology H , vertices in an intermediate stage i have exactly two incident edges to vertices in stage $i + 1$, and exactly two incident edges to vertices in stage $i - 1$. This implies that in the resulting configured ring network, each node has channel degree at most two.

The Benes topology has the property that for any permutation $\pi(\cdot)$ on $\{0, \dots, W-1\}$, there is an interconnection instance $(\tau(\cdot), h_0, \dots, h_{W-1})$ such that $\tau(\cdot)$ satisfies $(\tau(0), \tau(1), \dots, \tau(W-1)) = (0, 1, \dots, W-1)$, i.e., $\tau(\cdot)$ is the *identity* function. Thus, for $i = 0, \dots, W-1$, the path h_i starts at vertex u_i and ends at vertex $v_{\pi(i)}$. The Benes topology is referred to as a

permutation network since it has this property. Fig. 3 shows an example $\{h_0, h_1, h_2, h_3\}$ for the permutation $\pi(\cdot)$ that satisfies $(\pi(0), \pi(1), \pi(2), \pi(3)) = (1, 2, 3, 0)$ for the case when $W = 4$. Here,

$$h_0 = u_0 - x_0(1) - x_1(2) - x_0(3) - v_1$$

$$h_1 = u_1 - x_0(1) - x_0(2) - x_1(3) - v_2$$

$$h_2 = u_2 - x_1(1) - x_1(2) - x_1(3) - v_3,$$

and

$$h_3 = u_3 - x_1(1) - x_0(2) - x_1(3) - v_2.$$

As an example of a network configuration consider a 4-node ring network with $W = 4$ channels per link. Let H be the Benes network graph in Fig. 3. The edges of H between the stage 0 and stage 1 vertices are assigned to the channels of link 0. Similarly, the edges between stages 1 and 2 are assigned to the channels of link 1, the edges between stages 2 and 3 are assigned to the channels of link 2, and the edges between stages 3 and 4 are assigned to the channels of link 3. In the figure, the channel numbers for each edge are given. For example, edge $x_0(1) - x_1(2)$ is assigned to a channel numbered 1 (in link 1), i.e., $\gamma(x_0(1) - x_1(2))$ is the channel numbered 1 in link 1. Notice that vertices u_0, u_1, u_2 , and u_3 are assigned to channels numbered 0, 1, 2, and 3, respectively. Also, vertices v_0, v_1, v_2 , and v_3 are assigned to channels numbered 0, 1, 2, and 3, respectively. Now, if a pair of edges of H are incident to a common vertex in stage i ($i = 1, \dots, s - 1$) and one edge is between

stages $i - 1$ and i and the other is between stages i and $i + 1$ then their assigned channels are attached through node i . For example, edges $x_0(1) - x_1(2)$ and $x_1(2) - x_1(3)$ of H are incident to a common vertex $x_1(2)$. Then their associated channels in the ring network (channel 1 of link 1 and channel 3 of link 2) are attached through node 2. Note that node 0 has no channel conversion. The corresponding wiring arrangement for the ring network configuration is shown in Fig. 4. Nodes 1, 2 and 3 realize a Benes network graph and node 0 is wired so that there is no channel conversion.

Once the ring network has been configured (with respect to some H), then a channel assignment can be found for any request that satisfies $\lambda_{\max} \leq W$. We will now describe a channel assignment for such a request $\{p_1, \dots, p_m\}$, where m is the number of routes in the request.

First, a cut-and-color partition (P_0, \dots, P_{W-1}) is found for the request. Next, a permutation $\pi(\cdot)$ on $\{0, 1, \dots, W - 1\}$ is found with the following property: for each cut route p_i of the request, consider its left residual path a_i and right residual path b_i , and if the a_i is in P_j and b_i is in P_k then $\pi(j) = k$. We will refer to such a permutation as a *permutation for the cut-and-color partition*. (Note that there may be more than one permutation for a partition if the number of cut paths is less than W .)

One method to determine a permutation $\pi(\cdot)$ of the cut-and-color partition is as follows. Let Γ denote a set that equals $\{0, \dots, W - 1\}$. Now for each cut route p_i , of the request do the following: (1) determine the left residual path a_i and right residual path b_i of p_i ; (2) determine j_a and j_b such that $a_i \in P_{j_a}$ and $b_i \in P_{j_b}$; and then let $\pi(j_a) = j_b$ and remove the value j_b from the set Γ . For each $i = 0, \dots, W - 1$, such that the value of $\pi(i)$ has yet

to be determined, pick a value j from Γ , and then let $\pi(i) = j$ and remove j from Γ . For example, suppose $W = 4$ and the only cut routes of the request are p_1 and p_2 . Suppose the cut-and-color partition (P_0, \dots, P_3) is such that $a_1 \in P_2$, $b_1 \in P_3$, $a_2 \in P_3$, and $b_2 \in P_0$. Then $\pi(2) = 3$ and $\pi(3) = 0$. This leaves the values of $\pi(0)$ and $\pi(1)$ yet to be determined. Their values should not be from the set $\{0, 3\}$, which have already been used. Thus, we can let $\pi(0) = 2$ and $\pi(1) = 1$ which will leave $\pi(\cdot)$ a permutation.

Now for each $i = 0, \dots, W - 1$, a collection of channels of the ring network is assigned to P_i , one channel per link of the ring network. This is done as follows. For the graph H and permutation $\pi(\cdot)$ (of the cut-and-color partition), find the interconnection instance $(\tau(\cdot), h_0, h_1, \dots, h_{W-1})$. For each $i = 0, \dots, W - 1$, let

$$\{e_i(0), e_i(1), \dots, e_i(j), \dots, e_i(s-2)\}$$

denote the edges of H traversed by path $h_{\tau(i)}$, where $e_i(j)$ is the one between stages j and $j + 1$. Let

$$\{g_i(0), g_i(1), \dots, g_i(j), \dots, g_i(s-2)\}$$

be the collection of channels of the ring network, where $g_i(j)$ is the channel assigned to edge $e_i(j)$. i.e., $g_i(j) = \gamma(e_i(j))$. In addition, if $s \leq N$ then let

$$\{g_i(s-1), g_i(s), g_i(s+1), \dots, g_i(j), \dots, g_i(N-1)\}$$

be the collection of channels of the ring network, where $g_i(j)$ is the channel numbered $\tau(\pi(i))$

of link j . The collection

$$\{g_i(0), g_i(1), \dots, g_i(N-1)\}$$

are the channels assigned to P_i .

The channel assignment for the request can now be determined. For each uncut route p_i , assign channels to it as follows. Find k such that $p_i \in P_k$. For each link j of the ring network traversed by p_i , assign channel $g_i(j)$ to route p_i .

For each cut route p_i , assign channels to it as follows. Let a_i and b_i be the residual paths of p_i . Find k_a and k_b such that $a_i \in P_{k_a}$ and $b_i \in P_{k_b}$. For each link j traversed by a_i , assign channel $g_{k_a}(j)$ to route p_i . For each link j traversed by b_i , assign channel $g_{k_b}(j)$ to route p_i .

Example: As an example consider a 4-node ring network with $W = 4$ channels per link and configured according to the H in Fig. 3. The corresponding wiring arrangement for the ring network configuration is shown in Fig. 4. Nodes 1, 2 and 3 realize a Benes interconnection network and node 0 is wired so that there is no conversion.

Consider the same request as in Fig. 2. The cut-and-color partition is the same as before. A permutation $\pi(\cdot)$ for the partition is

$$(\pi(0), \pi(1), \pi(2), \pi(3)) = (1, 2, 3, 0).$$

(Notice, since there are only two cut routes in the request $\{p_0, \dots, p_5\}$, that there are other permutations for the partition, e.g., $(\pi'(0), \pi'(1), \pi'(2), \pi'(3)) = (1, 2, 0, 3)$.)

An interconnection instance $(\tau(\cdot), h_0, h_1, h_2, h_3)$ for $\pi(\cdot)$ is where $\tau(\cdot)$ is the identity function (i.e., $(\tau(0), \tau(1), \tau(2), \tau(3)) = (0, 1, 2, 3)$) and

$$h_0 = u_0 - x_0(1) - x_1(2) - x_0(3) - v_1,$$

$$h_1 = u_1 - x_0(1) - x_0(2) - x_1(3) - v_2,$$

$$h_2 = u_2 - x_1(1) - x_1(2) - x_1(3) - v_3,$$

and

$$h_3 = u_3 - x_1(1) - x_0(2) - x_0(3) - v_0,$$

as shown in Fig. 3. Equivalently, the paths traverse the following edges of H :

$$h_0 : u_0 - x_0(1), x_0(1) - x_1(2), x_1(2) - x_0(3), x_0(3) - v_1,$$

$$h_1 : u_1 - x_0(1), x_0(1) - x_0(2), x_0(2) - x_1(3), x_1(3) - v_2,$$

$$h_2 : u_2 - x_1(1), x_1(1) - x_1(2), x_1(2) - x_1(3), x_1(3) - v_3,$$

$$h_3 : u_3 - x_1(1), x_1(1) - x_0(2), x_0(2) - x_0(3), x_0(3) - v_0.$$

Using the assignment of edges to channels, as shown in Fig. 3, we can get an assignment of channels to each P_i ($i = 0, 1, 2, 3$). For example, for P_0 , we consider the edges traversed by h_0 . The edge $u_0 - x_0(1)$ is assigned to channel 0 in link 0, the edge $x_0(1) - x_1(2)$ is assigned to channel 1 in link 1, the edge $x_1(2) - x_0(3)$ is assigned to channel 1 in link 2, and the edge $x_0(3) - v_1$ is assigned to channel 1 in link 3. The following are the channel assignments to each P_i ($i = 0, 1, 2, 3$).

Set	Channels			
	Link 0	Link 1	Link 2	Link 3
P_0	0	1	1	1
P_1	1	0	2	2
P_2	2	3	3	3
P_3	3	2	0	0

Based on this, the individual routes are assigned channels. For example, consider an uncut route $p_3 = 2 - 3 - 0$. Notice that $p_3 \in P_2$, and so p_3 uses channels assigned to P_2 . Since p_3 traverses links 2 and 3, its channels are (according to the table above) channel 3 in link 2 and channel 3 in link 3. As another example, consider the cut route $p_2 = 2 - 3 - 0 - 1$. Notice that p_2 has the residual paths $a_2 = 2 - 3 - 0$ and $b_2 = 0 - 1$. Notice that $a_2 \in P_0$, and so p_2 uses some of the channels assigned to P_0 . In particular, since a_2 traverses links 2 and 3, the channels are (according to the table above) channel 1 in link 2 and channel 1 in link 3. Notice that $b_2 \in P_1$, and so p_2 uses a channel assigned to P_1 . In particular, since b_2 traverses link 0, the channel is (according to the table above) channel 1 in link 0.

The channel assignment for the request $\{p_0, \dots, p_5\}$ is shown in the table below.

Route	Links			
	0	1	2	3
p_0	0	1	-	-
p_1	-	0	2	-
p_2	1	-	1	1
p_3	-	-	3	3
p_4	2	3	-	2
p_5	-	2	0	-

The switching arrangement in the line cards to do this is shown in Fig. 5.

Configuration for Channel Degree $\Delta + 1$, where $\Delta > 1$

Consider a ring network with $N \geq \log_{\Delta} W$ nodes. There is a configuration that has channel degree at most $\Delta + 1$ at each node with the following property. All requests that have load at most W are feasible.

Consider the following network configuration. For each link $i = 0, 1, \dots, N-1$, its channel $j \in \{0, 1, \dots, W-1\}$ is attached to the following channels on link $(i+1) \bmod N$: channel $(j+1) \bmod W$ and channels $\{(j - k \cdot \Delta^i) \bmod W : k = 0, 1, \dots, \Delta-1\}$. Note that in this configuration, each node has channel degree at most $\Delta + 1$.

As an example consider the case of a 4-node ring network with $W = 4$ channels per link, and $\Delta = 2$. Then for each link $i \in \{0, 1, 2, 3\}$, its channel $j \in \{0, 1, 2, 3\}$ is attached to channels $(j+1) \bmod 4$, j , and $(j - 2^i) \bmod 4$ on link $(i+1) \bmod 4$. For example, channel 1 on link 0 is attached to channels 2, 1, and 0 on link 1. As another example, note that channel 2 on link 3 is attached to channels 3 and 2 on link 0. The wiring arrangement is

shown in Fig. 6.

Now consider an arbitrary request $\{p_1, \dots, p_m\}$ with load at most W . We will now describe how to find a channel assignment for it. We can find a cut-and-color partition (P_0, \dots, P_{W-1}) and a permutation $\pi(\cdot)$ for the partition as before. We will use the following definition. We call two numbers i and j in $\{0, 1, \dots, W\}$ to be π -related if there is a value k and a sequence (r_0, r_1, \dots, r_k) of numbers from $\{0, \dots, W-1\}$ such that $r_0 = i$, $r_k = j$, and for $i = 0, 1, \dots, k-1$, $\pi(r_i) = r_{i+1}$. For example, suppose $W = 8$ and

$$(\pi(0), \pi(1), \pi(2), \pi(3), \pi(4), \pi(5), \pi(6), \pi(7)) = (1, 3, 7, 5, 4, 0, 2, 6).$$

Note that $\pi(0) = 1$, $\pi(1) = 3$, $\pi(3) = 5$, and $\pi(5) = 0$. Thus, the numbers $\{0, 1, 3, 5\}$ are π -related. Similarly, the numbers within the following subsets are π -related: $\{2, 7, 6\}$ and $\{4\}$.

Partition the set $\{0, \dots, W-1\}$ into nonempty subsets $\{C_0, \dots, C_{M-1}\}$, where M is the number of subsets, such that numbers within a subset are π -related, while numbers from different subsets are not. Continuing with our example, the subsets could be $C_0 = \{0, 1, 3, 5\}$, $C_1 = \{2, 7, 6\}$, and $C_2 = \{4\}$. For each $i = 0, \dots, M-1$, let s_i denote the size of C_i . Then for the example, $s_0 = 4$, $s_1 = 3$, and $s_2 = 1$.

Define any subset of $\{0, \dots, W-1\}$ as a *contiguous* subset if it can be written as

$$\{(i+j) \bmod W : j = 0, \dots, k\}$$

for some i and k in $\{0, \dots, W-1\}$. Partition $\{0, \dots, W-1\}$ into W contiguous subsets

(T_0, \dots, T_{M-1}) such that T_i has size s_i . This can be done by finding a collection of numbers $\{t_0, \dots, t_{M-1}\}$ from $\{0, \dots, W-1\}$ such that for $i = 0, \dots, M-1$,

$$t_{(i+1) \bmod M} = (t_i + s_i) \bmod W.$$

Then for $i = 0, \dots, M-1$,

$$T_i = \{(t_i + j) \bmod W : j = 0, \dots, s_i - 1\}.$$

To continue with our example, we could have $t_0 = 0$, $t_1 = 4$, $t_2 = 7$, $T_0 = \{0, 1, 2, 3\}$, $T_1 = \{4, 5, 6\}$, and $T_2 = \{7\}$.

For $i = 0, \dots, M-1$, find a function $q_i(\cdot)$ that is defined on the set $\{0, \dots, s_i - 1\}$ such that

1. there is an element $j \in C_i$ such that $q_i(j) = 0$ and
2. for each element $j \in C_i$, $q_i(\pi(j)) = (q_i(j) + 1) \bmod s_i$.

To continue with our example, let us determine what $q_0(\cdot)$ should be. Recall that $C_0 = \{0, 1, 3, 5\}$, and that $\pi(0) = 1$, $\pi(1) = 3$, $\pi(3) = 5$, and $\pi(5) = 0$. Then we could have $(q_0(0), q_0(1), q_0(3), q_0(5)) = (0, 1, 2, 3)$. Similarly, we could have $(q_1(2), q_1(7), q_1(6)) = (0, 1, 2)$, and $(q_2(4)) = (0)$.

For $k = 0, \dots, M-1$, let $(d_{N-1}(k), d_{N-2}(k), \dots, d_0(k))$ denote the base Δ , N digit representation of the value $s_k - 1$. Now, for $i = 0, \dots, N-1$, let

$$D_i(k) = \begin{cases} 0, & \text{if } i = 0 \\ \sum_{n=0}^{i-1} d_n(k) \cdot \Delta^n, & \text{if } i > 0 \end{cases}$$

For example, if $N = 4$, $s_k - 1 = 15$, and $\Delta = 2$ then

$$(d_3(k), d_2(k), d_1(k), d_0(k)) = \text{the binary number } (1, 1, 1, 1),$$

and

$$(D_3(k), D_2(k), D_1(k), D_0(k)) = (7, 3, 1, 0).$$

As another example, if $N = 3$, $s_k - 1 = 15$, and $\Delta = 3$ then

$$(d_2(k), d_1(k), d_0(k)) = \text{the ternary number } (1, 2, 0),$$

and

$$(D_2(k), D_1(k), D_0(k)) = (6, 0, 0).$$

For each subset P_i ($i = 0, \dots, W-1$) from the cut-and-color partition, we assign it channels as follows. The channels assigned to P_i will be denoted by $\sigma(i, 0), \sigma(i, 1), \dots, \sigma(i, j), \dots, \sigma(i, N-1)$, where $\sigma(i, j)$ is the channel on link j . Let k be such that $P_i \in C_k$. For $j = 0, \dots, N-1$, let $\rho(i, j)$ be the following value

$$\rho(i, j) = \begin{cases} s_k - 1 - D_j(k), & \text{if } q_k(i) = s_k - 1 \\ q_k(i), & \text{if } q_k(i) < s_k - 1 \text{ and } q_k(i) < s_k - 1 - D_j(k) \\ q_k(i) + 1, & \text{if } q_k(i) < s_k - 1 \text{ and } q_k(i) \geq s_k - 1 - D_j(k) \end{cases}$$

For $j = 0, \dots, N-1$, let $\sigma(i, j) = (t_k + \rho(i, j)) \bmod W$. For example, suppose $N = 4$, $\Delta = 2$,

$W = 32$, and $C_k = \{4, 5, \dots, 11\}$. Here, note that $s_k = 8$,

$$(d_3(k), d_2(k), d_1(k), d_0(k)) = (0, 1, 1, 1),$$

and

$$(D_3(k), D_2(k), D_1(k), D_0(k)) = (7, 3, 1, 0).$$

Suppose that

$$(\pi(4), \pi(5), \dots, \pi(11)) = (5, 6, \dots, 11, 4)$$

and

$$(q_k(4), q_k(5), \dots, q_k(11)) = (0, 1, \dots, 6, 7).$$

In addition, to simplify the example, suppose that $t_k = 0$, so that $\sigma(i, j) = \rho(i, j)$ for all $i \in C_k$. Then we have the following channel assignment for the subsets in C_k :

Link	Sets							
	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}
0	0	1	2	3	4	5	6	7
1	0	1	2	3	4	5	7	6
2	0	1	2	3	5	6	7	4
3	1	2	3	4	5	6	7	0

The values of $\sigma(l, j)$, where $l \in C_k$, can be read from the table. For example, the channels assigned to P_8 are channel $\sigma(8, 0) = 4$ in link 0, channel $\sigma(8, 1) = 4$ in link 1, channel $\sigma(8, 2) = 5$ in link 2, and channel $\sigma(8, 3) = 5$ in link 3. To see what the table looks like when t_k is not zero, suppose the t_k were changed to 10. Then the following channel assignment

for the subsets in C_k would result.

	Sets							
Link	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}
0	10	11	12	13	14	15	16	17
1	10	11	12	13	14	15	17	16
2	10	11	12	13	15	16	17	14
3	11	12	13	14	15	16	17	10

Channels can be assigned to each route p_k of the request as follows. Suppose p_k is an uncut route. Let i be such that $p_k \in P_i$. For each link j that is traversed by p_k , the channel $\sigma(i, j)$ of the link is assigned to p_k . Suppose p_k is a cut route. Let a_k and b_k be its residual paths. Let i_a and i_b be such that $a_k \in P_{i_a}$ and $b_k \in P_{i_b}$. For each link j that is traversed by a_k , the channel $\sigma(i_a, j)$ of the link is assigned to p_k . For each link j that is traversed by b_k , the channel $\sigma(i_b, j)$ of the link is assigned to p_k .

Example: Consider a 4-node ring network that has $W = 4$ channels per link, and where it is configured according to $\Delta = 2$. Hence, the wiring arrangement in the line cards is shown in Fig. 6.

Suppose the requests are shown in Fig. 2. The cut-and-color partition and the permutation $\pi(\cdot)$ for the partition is the same as before. Thus, $(\pi(0), \pi(1), \pi(2), \pi(3)) = (1, 2, 3, 0)$. Then we have $C_0 = \{0, 1, 2, 3\}$, $s_0 = 4$,

$$(d_3(0), d_2(0), d_1(0), d_0(0)) = (0, 0, 1, 1),$$

$$(D_3(0), D_2(0), D_1(0), D_0(0)) = (3, 3, 1, 0),$$

and

$$(q_0(0), q_0(1), q_0(2), q_0(3)) = (0, 1, 2, 3)$$

Thus the sets P_0, P_1, P_2, P_3 are assigned channels on the links as follows:

Set	Links			
	0	1	2	3
P_0	0	0	1	1
P_1	1	1	2	2
P_2	2	3	3	3
P_3	3	2	0	0

Based on this, the individual routes are assigned channels as given below:

Route	Links			
	0	1	2	3
p_0	0	0	-	-
p_1	-	1	2	-
p_2	1	-	1	1
p_3	-	-	3	3
p_4	2	3	-	2
p_5	-	2	0	-

The switch settings corresponding to this assignment are shown in Fig. 7.

Star Network

Fig. 8 shows the block diagram of multiplexors 101 connected in a star network configuration. The network consists of a *hub* node 102H and *spoke* nodes 102E. The spoke nodes are connected to the hub node by a transmission link or medium 103. Each spoke node 102E in the network consists of a multiplexor. The hub node consists of a multiplexor for each link (or each spoke node) in the network. The multiplexors in the hub node are wired together according to a specified pattern. The figure shows 4 channels on each link. For each channel there is a line card 104 within each multiplexor. A line card consists of an I/O port 105, multiple local ports 106 and a line port 107 and a switch (not shown in the figure) that allows any pairs of these ports to be connected together.

Our results use the following network configuration of channels when W , the number of channels per link, is even. Each link has its channel $i = 0, 1, \dots, W/2 - 1$ connected to channel $w(i)$ (through the hub node) on all the other links, where $w(i) = i + W/2$. We will denote the hub node by h , and the spoke nodes by x_1, \dots, x_{N-1} . For $i = 1, \dots, N - 1$, let e_i denote the link between nodes h and x_i .

Once the network is configured, a channel assignment may be found for any request that has load at most W and each route of the request traverses at most two links. The following is the procedure to find a channel assignment. Let $\{p_1, \dots, p_M\}$ denote the routes of the request. Let $\{p_1, \dots, p_m\}$ denote the routes that traverse exactly two links. Hence, the routes $\{p_{m+1}, \dots, p_M\}$ denote the ones that traverse exactly one link.

We will refer to a path as being *incident* to its end nodes. For example, a path that traverses a sequence of nodes (x_i, h, x_j) (hence, it traverses exactly two links), is considered

to be incident to its end nodes x_i and x_j (here, h is an *intermediate* node). As another example, a path that traverses the sequence of nodes (x_i, h) (hence, it traverses exactly one link), is considered to be incident to its end nodes x_i and h .

A path may be *directed*, which means that it is viewed as going from one of its end nodes to its other end node. For example, if a path traverses two links and has end nodes x_i and x_j , then it may be *directed* from x_i to h and then to x_j , or it may be *directed* from x_j to h and then to x_i . If a path traverses one link and has end nodes x_i and h then it may be directed from x_i to h , or it may be directed from h to x_i . As part of the channel assignment procedure, the routes $\{p_1, \dots, p_m\}$ will be directed so that at each spoke node there are at most $W/2$ incident routes of $\{p_1, \dots, p_m\}$ that are directed into the node, and at most $W/2$ incident routes of $\{p_1, \dots, p_m\}$ that are directed out of the node. The procedure to direct these routes is as follows.

If the number of routes of $\{p_1, \dots, p_M\}$ that traverse each link is exactly W then let $R = M$. Otherwise, find additional paths $\{p_{M+1}, \dots, p_R\}$ such the number of routes of $\{p_1, \dots, p_R\}$ that traverse each link is exactly W . The additional paths $\{p_{M+1}, \dots, p_R\}$ are referred to as *dummy* paths. Note that the dummy paths can be found as follows. For $i = 1, \dots, N - 1$, let there be $W - n_i$ dummy paths, each traversing only link e_i , where n_i is the number of routes (that are not dummy paths) traversing link e_i .

The paths of $\{p_1, \dots, p_R\}$ are directed as follows. Consider each path of $\{p_1, \dots, p_R\}$ as being initially undirected. Refer to a node that has at least one undirected incident path as a *free* node. As long as there is a free node, do the following:

1. Start from a free node, say x_i , and traverse an undirected incident path (from the set

- $\{p_1, \dots, p_R\}$) to the other end node, and direct the path in the direction of the traversal.
2. From the other end node, traverse an undirected incident path (from the set $\{p_1, \dots, p_R\}$) to the next end node, and direct the path in the direction of the traversal.
 3. Keep traversing undirected paths (and directing the traversed paths) in this way until node x_i is reached.

Now construct a bipartite graph G which has two sets of vertices: $\{u_1, \dots, u_{N-1}\}$ and $\{v_1, \dots, v_{N-1}\}$. It has edges b_1, \dots, b_m , where b_i is between u_j and v_k if path p_i traverses links e_j and e_k in the star network and p_i is directed so that it goes from node x_j to h and then to x_k . Note that in G , each vertex has at most $W/2$ incident edges because each spoke node of the star network has at most $W/2$ incoming incident paths and at most $W/2$ outgoing incident paths. Next, assign numbers $\{0, \dots, W/2 - 1\}$ to the edges of G such that distinct numbers are assigned to edges incident to a common node, and denote the number assigned to link b_i (for $i = 1, \dots, m$) by $q(b_i)$. This can be accomplished using the scheduling algorithms used for Satellite Switched/Time Division Multiple Access (SS/TDMA) systems [13], incorporated herein by reference. Using the assignment of numbers, we can get a channel assignment for the routes $\{p_1, \dots, p_m\}$ as follows. For $i = 1, \dots, m$, suppose p_i traverses links e_j and e_k such that the direction of p_i goes from x_j to h and then to x_k . Then channel $q(b_i)$ on link e_j is assigned to p_i , and the channel $w(q(b_i))$ on link e_k is also assigned to p_i .

Note that up to this point, channels have been assigned to the routes $\{p_1, \dots, p_m\}$. Now channels will be assigned to the routes $\{p_{m+1}, \dots, p_M\}$ (i.e., the routes that traverse exactly one link). This can be done by selecting each route and assigning it a channel on the link that it traverses that has yet to be assigned to a route.

Example: Consider the five node star network of Fig. 8, redrawn in Fig. 9(A). The network has a hub node h , and four spoke nodes $\{x_1, x_2, x_3, x_4\}$. Note that for $i = 1, 2, 3, 4$, spoke node x_i and hub node h have link e_i between them. Note that each link has $W = 4$ channels numbered $0, 1, 2, 3$. These channel numbers are partitioned into two groups: $\{0, 1\}$ and $\{2, 3\}$. Note that $w(0) = 2$ and $w(1) = 3$. The hub node is configured so that for $i = 0, 1$, a channel i at each link is connected to channel $w(i)$ at all the other links.

Now suppose there is a request $\{p_1, p_2, \dots, p_6\}$ of six routes as shown in Fig. 9(A). These routes are as follows:

$$p_1 = x_1 - h - x_2$$

$$p_2 = x_2 - h - x_3$$

$$p_3 = x_3 - h - x_1$$

$$p_4 = x_1 - h - x_4$$

$$p_5 = x_3 - h - x_4$$

and

$$p_6 = x_3 - h - x_1.$$

Note that there are $W = 4$ routes of the request traversing links e_1 and e_3 , but there are only two routes of the request traversing links e_2 and e_4 . Dummy paths p_7, p_8, p_9 , and p_{10} are found for the links e_2 and e_4 as shown in Fig. 9(A). Note that the paths p_7 and p_8 only traverse link e_2 , and paths p_9 and p_{10} only traverse link e_4 . Now each link has exactly $W = 4$ paths traversing it.

Paths p_1, \dots, p_{10} are initially considered *undirected*. Then they are *directed* as follows. First a node is chosen that has an undirected path incident to it (i.e., a free node is chosen). Node x_1 is such a node since it has undirected paths p_1, p_3, p_4, p_6 incident to it. One of the undirected incident paths is chosen to be traversed, say path p_1 . After traversing it to node x_2 , it is directed from end node x_1 to end node x_2 . From node x_2 , an undirected incident path is chosen to be traverse. Such paths are p_2, p_7, p_8 . Suppose path p_2 is chosen. After traversing it to node x_3 , it is directed from end node x_2 to end node x_3 . From node x_3 , an undirected incident path is chosen to be traversed. Such paths are p_3, p_5, p_6 . Suppose path p_3 is chosen. After traversing it to node x_1 , it is directed from end node x_3 to end node x_1 . Note that the paths p_1, p_2, p_3 are directed as shown in Fig. 9(B). Since we returned to node x_1 , we start the procedure of directing paths all over again. Fig. 9(B) shows the direction of paths p_4, p_5, p_6 which results by starting from node x_4 and traversing paths p_5, p_6 , and then p_4 . Fig. 9(B) also shows the direction of paths p_7, p_8, p_9, p_{10} which results by starting from node x_2 and traversing paths p_7, p_9, p_{10} , and then p_8 . Note that we have the following directions for the paths:

$$p_1 = x_1 \rightarrow h \rightarrow x_2$$

$$p_2 = x_2 \rightarrow h \rightarrow x_3$$

$$p_3 = x_3 \rightarrow h \rightarrow x_1$$

$$p_4 = x_1 \rightarrow h \rightarrow x_4$$

$$p_5 = x_4 \rightarrow h \rightarrow x_3$$

$$p_6 = x_3 \rightarrow h \rightarrow x_1$$

$$p_7 = x_2 \rightarrow h$$

$$p_8 = h \rightarrow x_2$$

$$p_9 = h \rightarrow x_4$$

and

$$p_{10} = x_4 \rightarrow h.$$

We now construct a bipartite graph G , as shown in Fig. 9(C), with two sets of vertices $\{u_1, u_2, u_3, u_4\}$ and $\{v_1, v_2, v_3, v_4\}$. There are six edges between the nodes denoted by $\{b_1, b_2, \dots, b_6\}$. For $i = 1, \dots, 6$, the edge b_i corresponds to the route p_i in the request. If p_i has end nodes x_j and x_k and is directed from x_j to x_k then edge b_i is between vertices u_j and v_k . Thus, the edges of G are

$$b_1 = u_1 - v_2$$

$$b_2 = u_2 - v_3$$

$$b_3 = u_3 - v_1$$

$$b_4 = u_1 - v_4$$

$$b_5 = u_4 - v_3$$

and

$$b_6 = u_3 - v_1.$$

Numbers from the set $\{0, 1\}$ (i.e., $\{0, \dots, W/2 - 1\}$) are assigned to the edges of G so that at each vertex of G , its incident edges have distinct numbers. The number assigned to edge b_i will be denoted by $q(b_i)$. A number assignment is shown in Fig. 9(C). Here, $q(b_1) = 0$, $q(b_2) = 1$, $q(b_3) = 0$, $q(b_4) = 1$, $q(b_5) = 0$, and $q(b_6) = 1$. Note that the SS/TDMA scheduling algorithm can be used to determine $q(b_i)$ for each edge b_i of G .

The channel assignment to the routes are as follows. Note that p_1 corresponds to b_1 , which has end vertices u_1 and v_2 . Note that u_1 corresponds to link e_1 , and v_2 corresponds to link e_2 . The channels assigned to p_1 are channel $q(b_1) = 0$ on link e_1 and channel $w(q(b_1)) = 2$ on link e_2 . The channel assignment for all the routes of the request are given below:

p_1 : channel 0 on link e_1 , and channel 2 on link e_2 ,

p_2 : channel 1 on link e_2 , and channel 3 on link e_3 ,

p_3 : channel 0 on link e_3 , and channel 2 on link e_1 ,

p_4 : channel 1 on link e_1 , and channel 3 on link e_4 ,

p_5 : channel 0 on link e_4 , and channel 2 on link e_3 ,

and

p_6 : channel 1 on link e_3 , and channel 3 on link e_1 .

The corresponding setting of the switches and channel assignment in the network are shown in Fig. 10 for routes p_1, p_2 and p_3 as an illustration.

Arbitrary Topology Networks

Consider an arbitrary topology network such that each link has W channels, where W is even. Then the following method gives a fixed conversion configuration of the network and a channel assignment that assigns channels for any set of connections with routes that have congestion at most W and have at most two hops.

The channel assignment is done by converting the given network into a star network as follows. Each link i' in the star network corresponds to a link i in the original network. A connection that is to be routed on links i and j in the original network is now to be routed on links i' and j' in the star network. The congestion in the star network is at most W and hence these connections can be routed using the results of the star configuration.

Claims

1. In a ring communications network having N nodes where each link between said nodes is a multichannel multiplexed link having W channels, denoted by channel numbers $\{0, 1, \dots, W - 1\}$ a method of configuring said nodes of said network, said method comprising:
 - (a) designating one of said nodes as a primary node;
 - (b) configuring said primary node so that any two channels between its two incident

- links can be connected to each other; and
- (c) configuring each of said nodes which is not a primary node so that a channel c on one of its incident links can be connected to the same channel c on its other incident link.
2. In a ring communications network having a plurality of channels on each of a plurality of links interconnecting nodes of said network, with one of said nodes being a primary node which is configured so that any two channels between its incident links can be connected to each other, wherein said channels of said links are assigned to a set of end-to-end communication connections $1, \dots, j, \dots, m$, where each end-to-end communication connection j is a sequence of connected channels that follow a path p_j on said ring communications network, a method of assigning channels to said paths $\{p_1, \dots, p_m\}$ comprising:
- (a) identifying each path p_i as a cut path if it passes through said primary node and as an uncut path if it does not pass through said primary node;
- (b) for each cut path p_i , forming two paths a_i and b_i , to be referred to as residual paths, by splitting p_i into two at said primary node such that said primary node becomes an end node for both said residual paths;
- (c) assigning a single channel $c(p_j)$ to each uncut path p_j , and assigning a single channel $c(a_i)$ and a single channel $c(b_i)$ to each residual path a_i and b_i , respectively, wherein each connection using one of said paths is assigned a channel on each link of its path where no two of said connections are assigned the same channel on the

same link;

(d) for each uncut path p_i , assigning to it said channel $c(p_i)$ of each link that it traverses; and

(e) for each cut path p_i , assigning to it the said channel $c(a_i)$ of each link that residual path a_i traverses, and assign it channel $c(b_i)$ of each link that residual path b_i traverses.

3. A ring communications network for providing high utilization of its bandwidth, said network comprising: a plurality of nodes connected to each other by a plurality of links, with one of said nodes being a primary node capable of connecting any two channels between its two incident links, of said links, and with each of the remaining ones of said nodes capable of connecting only the same channel to each other on its two incident links, of said links, wherein a circuit connection terminating at two of said nodes is established by assigning channels on those of said links that are used for a path of said connection.

4. In a communications ring network having a plurality of nodes interconnected to each other by a plurality of links, a method of assigning channels on each link, of said links, along paths of connections through said network, said method comprising:

(a) providing full channel connectivity between any two channels of the two incident links, of said links, of a primary node, of said nodes;

(b) for each of the remaining ones of said nodes, attaching only the same channels to each other on its incident links;

- (c) for each of said connections, assigning a channel on each link of its path such that no two connections use the same channel on a common link, of said links;
 - (d) for each connections whose path does not pass through said primary node, assigning the same single channel in each of said links along its entire path;
 - (e) identifying all paths of said connections that pass through said primary node, where latter said paths are referred to as cut paths;
 - (f) for each cut path, forming two paths, referred to as residual paths, by splitting said each cut path at said primary node which becomes an end node for said two residual paths of said each cut path; and
 - (g) assigning a single channel to each one of said residual paths.
5. In a ring communications network having N nodes, wherein each link between nodes is a multichannel multiplexed link having W channels, denoted by channel numbers $\{0, 1, \dots, W - 1\}$, a method of configuring said nodes of said network, said method comprising:
- (a) designating one of said nodes as a primary node and numbering said nodes $0, 1, \dots, N - 1$ starting from the primary node and proceeding in only one direction around said ring to an adjacent one of said nodes;
 - (b) for $i = 0, \dots, N - 1$, numbering the link between the pair of nodes i and $i + 1 \bmod N$ with the number i ;
 - (c) creating graph H which is composed of a set of vertices and a set of edges, that go between pairs of vertices, where the set of vertices includes:

2025 RELEASE UNDER E.O. 14176

- i. a collection of W vertices $\{u_0, \dots, u_{W-1}\}$ called the stage 0 vertices,
- ii. a collection of W vertices $\{v_0, \dots, v_{W-1}\}$ called the stage s vertices, where $s \leq N + 1$, and
- iii. for each $i = 1, 2, \dots, s - 1$, a collection of vertices $\{x_0(i), x_1(i), \dots, x_{s_i-1}(i)\}$ called the stage i vertices, with s_i denoting the number of vertices in stage i , and

the set of edges includes:

- i. for each $i = 1, \dots, s - 2$, a collection of W edges between stage i vertices and stage $i + 1$ vertices,
 - ii. an edge from each vertex u_i in stage 0 to a vertex in stage 1, and
 - iii. an edge from a vertex in stage $s - 1$ to each vertex v_i in stage s ;
- (d) defining any function $f(\cdot)$ as a permutation if it is defined for the values $\{0, 1, \dots, W - 1\}$ such that $(f(0), f(1), \dots, f(W - 1))$ are distinct values from the set $\{0, 1, \dots, W - 1\}$;
- (e) said graph H having the property that for each permutation $\pi(\cdot)$, there is a permutation $\tau(\cdot)$ and a set of W paths $\{h_0, h_1, \dots, h_{W-1}\}$ in H such that:
- i. for each $i = 0, \dots, W - 1$, path h_i starts at node $u_{\tau(i)}$ in stage 0, traverses vertex stages $1, 2, \dots, s$ in succession, and ends at node $v_{\tau(\pi(i))}$ in stage s , and
 - ii. the paths $\{h_0, \dots, h_{W-1}\}$ do not have common edges in H ;
- the collection $(\tau(\cdot), h_0, h_1, \dots, h_{W-1})$ being referred to as an interconnection instance for $\pi(\cdot)$;

(f) assigning each edge e of said graph H to a channel, denoted by $\gamma(e)$, in the said ring communications network comprising:

- i. assigning the W edges between vertices of stages 0 and 1 in the said graph H to distinct channels in link 0 of the said ring communications network such that if edge e of H is incident to vertex u_i in stage 0, then e is assigned to channel i and $\gamma(e)$ equals i ,
- ii. assigning the W edges between vertices of stages $s-1$ and s in the said graph H to distinct channels in link $(s-1) \bmod N$ in the said ring communications network such that if edge e of H is incident to vertex v_i in stage s , then e is assigned to channel i and $\gamma(e)$ equals i , and
- iii. for each stage $i = 1, 2, \dots, s-2$, assigning the W edges between the vertices of stages i and $i+1$ in the graph H to distinct channels in link i of the said network, and for each of the said edges e of H , letting $\gamma(e)$ denote the channel of link i that e is assigned to;

(g) for $i = 1, 2, \dots, s-1$, configuring node $i \bmod N$ of said ring communications network such that channel c on link $(i-1) \bmod N$ is attached to channel c' on link $i \bmod N$ if:

- i. in the said graph H , there is an edge e between vertices of stages $i-1$ and i such that $\gamma(e) = c$,
- ii. in the said graph H , there is an edge e' between vertices of stages i and $i+1$ such that $\gamma(e') = c'$, and
- iii. in the said graph H , e and e' are incident to a common vertex in stage i ; and

(h) configuring the other nodes such that a connection on any channel c on one of its links can be connected to the same channel c on its other link.

6. A method as recited in claim 5, further comprising a method of assigning channels to a set of end-to-end communication connections $1, \dots, j, \dots, m$, where each end-to-end communication connection j is a sequence of connected channels that follow a path p_j on the said ring communications network, whose nodes are configured as recited in claim 5, said method further comprising:

- (a) identifying each path p_i as a cut path if it passes through said primary node and as an uncut path if it does not pass through said primary node;
- (b) for each cut path p_i , forming two paths a_i and b_i , referred to residual paths, by splitting p_i into two at the primary node, such that the primary node becomes an end node for both residual paths, and then labeling the residual path that traverses link 0 by b_i and labeling the residual path that traverses link $N - 1$ by a_i ;
- (c) partitioning the set of uncut and residual paths into W subsets denoted by (P_0, \dots, P_{W-1}) , such that in each subset no two paths traverse a common link;
- (d) finding a permutation $\pi(\cdot)$, as defined earlier in claim 5d, such that if p_j is a cut path, $a_j \in P_i$ and $b_j \in P_k$ then $\pi(i) = k$;
- (e) for the permutation $\pi(\cdot)$, finding an interconnection instance $(\tau(\cdot), h_1, h_2, \dots, h_{W-1})$, as defined in claims 5e, where recall that $\tau(\cdot)$ is a permutation and $\{h_0, \dots, h_{W-1}\}$ is a collection of paths in the said graph H of claim 5;

(f) for each uncut path p_i , assigning channels to it from each link it traverses such that:

i. if the path traverses a link j such that $j < s - 1$ then a channel c of the link is assigned to the path, where c is determined by:

A. finding the value k such that $p_i \in P_k$,

B. in the said graph H , finding the edge e traversed by the path $h_{\tau(k)}$ between the stage j vertices and the stage $j + 1$ vertices, and

C. letting c equal $\gamma(e)$,

ii. if the path traverses a link j such that $j \geq s - 1$ then a channel c of the link is assigned to the path, where c is determined by:

A. finding the value k such that $p_i \in P_k$,

B. letting c equal $\tau(\pi(k))$; and

(g) for each cut path p_i , assigning channels to it from each link it traverses such that:

i. if the path traverses a link j such that $j < s - 1$ and its residual path a_i (respectively, b_i) also traverses the link then a channel c of the link is assigned to the path p_i , where c is determined by

A. finding the value k such that $a_i \in P_k$ (respectively, $b_i \in P_k$),

B. in the said graph H , finding the edge e traversed by the path $h_{\tau(k)}$ between the stage j vertices and the stage $j + 1$ vertices, and

C. letting c equal $\gamma(e)$,

ii. if the path traverses a link j such that $j \geq s - 1$ and its residual path a_i

(respectively, b_i) also traverses the link then a channel c of the link is assigned to the path p_i , where c is determined by:

- A. finding the value k such that $a_i \in P_k$ (respectively, $b_i \in P_k$),
- B. letting c equal $\tau(\pi(k))$.

7. A method as recited in claim 5, wherein said graph H has the same topology as a $W \times W$ Benes network, which is composed of a set of vertices and a set of edges, that go between pairs of vertices, where

(a) the set of vertices includes:

- i. a collection of W vertices $\{u_0, \dots, u_{W-1}\}$ called the stage 0 vertices,
- ii. a collection of W vertices $\{v_0, \dots, v_{W-1}\}$ called the stage s vertices, where $s = 2n$ and $n = \log_2 W$,
- iii. for each $i = 1, 2, \dots, s - 1$, a collection of $W/2$ vertices $\{x_0(i), x_1(i), \dots, x_{W/2-1}(i)\}$ called the stage i vertices;

(b) and the set of edges includes:

- i. a collection of W edges between the stage 0 vertices and stage 1 vertices such that there is an edge between stage 0 vertex u_i and stage 1 vertex $x_j(1)$ if $(d_{n-1}, \dots, d_1) = (\delta_{n-2}, \dots, \delta_0)$, where (d_{n-1}, \dots, d_0) is the n bit binary representation of i and $(\delta_{n-2}, \dots, \delta_0)$ is the $n - 1$ bit binary representation of j ,
- ii. a collection of W edges between the stage $s - 1$ vertices and stage s vertices such that there is an edge between stage s vertex v_i and stage $s - 1$ vertex

$x_j(s-1)$ if $(d_{n-1}, \dots, d_1) = (\delta_{n-2}, \dots, \delta_0)$, where (d_{n-1}, \dots, d_0) is the n bit binary representation of i and $(\delta_{n-2}, \dots, \delta_0)$ is the $n-1$ bit binary representation of j ,

iii. a collection of W edges between stage 1 vertices and stage 2 vertices such that there is an edge between stage 1 vertex $x_i(1)$ and stage 2 vertex $x_j(2)$ if $(d_{n-2}, \dots, d_1) = (\delta_{n-3}, \dots, \delta_0)$, where (d_{n-2}, \dots, d_0) is the $n-1$ bit binary representation of i and $(\delta_{n-2}, \dots, \delta_0)$ is the $n-1$ bit binary representation of j ,

iv. a collection of W edges between stage $n-1$ vertices and stage n vertices such that there is an edge between stage $n-1$ vertex $x_i(n-1)$ and stage n vertex $x_j(n)$ if $(d_{n-2}, \dots, d_1) = (\delta_{n-2}, \dots, \delta_1)$, where (d_{n-2}, \dots, d_0) is the $n-1$ bit binary representation of i and $(\delta_{n-2}, \dots, \delta_0)$ is the $n-1$ bit binary representation of j ,

v. for each $k = 2, 3, \dots, n-2$, a collection of W edges between stage k vertices and stage $k+1$ vertices such that there is an edge between stage k vertex $x_i(k)$ and stage $k+1$ vertex $x_j(k+1)$ if $(d_{n-1-k}, \dots, d_1) = (\delta_{n-k-2}, \dots, \delta_0)$ and $(d_{n-2}, \dots, d_{n-k}) = (\delta_{n-2}, \dots, \delta_{n-k})$, where (d_{n-2}, \dots, d_0) is the $n-1$ bit binary representation of i and $(\delta_{n-2}, \dots, \delta_0)$ is the $n-1$ bit binary representation of j ,

vi. a collection of W edges between stage $s-2$ vertices and stage $s-1$ vertices such that there is an edge between stage $s-2$ vertex $x_i(s-2)$ and stage $s-1$ vertex $x_j(s-1)$ if $(d_{n-3}, \dots, d_0) = (\delta_{n-2}, \dots, \delta_1)$, where (d_{n-2}, \dots, d_0) is the

$n - 1$ bit binary representation of i and $(\delta_{n-2}, \dots, \delta_0)$ is the $n - 1$ bit binary representation of j ,

vii. a collection of W edges between stage n vertices and stage $n + 1$ vertices such that there is an edge between stage n vertex $x_i(n)$ and stage $n + 1$ vertex $x_j(n + 1)$ if $(d_{n-2}, \dots, d_1) = (\delta_{n-2}, \dots, \delta_1)$, where (d_{n-2}, \dots, d_0) is the $n - 1$ bit binary representation of i and $(\delta_{n-2}, \dots, \delta_0)$ is the $n - 1$ bit binary representation of j , and

viii. for each $k = n + 1, n + 2, \dots, s - 3$, a collection of W edges between stage k vertices and stage $k + 1$ vertices such that there is an edge between stage k vertex $x_i(k)$ and stage $k + 1$ vertex $x_j(k + 1)$ if $(d_{k-1-n}, \dots, d_0) = (\delta_{k-n}, \dots, \delta_1)$, and $(d_{n-2}, \dots, d_{k+1-n}) = (\delta_{n-2}, \dots, \delta_{k+1-n})$, where (d_{n-2}, \dots, d_0) is the $n - 1$ bit binary representation of i and $(\delta_{n-2}, \dots, \delta_0)$ is the $n - 1$ bit binary representation of j .

8. In a ring communications network having N nodes, wherein each link between each adjacent pair of said nodes is a multichannel multiplexed link, with W channels, denoted with channel numbers $\{0, \dots, W - 1\}$, a method of configuring each of said nodes of said network, said method comprising:

- (a) designating one node in a ring as the primary node and numbering the nodes 0, 1, ..., $N - 1$ starting from the primary node and proceeding in only one direction around said ring to an adjacent one of said nodes;
- (b) for $i = 0, \dots, N - 1$, numbering the link between nodes i and $(i + 1) \bmod N$ with the number i ; and

- (c) configuring each of said nodes such that channel c on link i may be connected to one of $\Delta + 1$ channels on link $(i + 1) \bmod N$, where $\Delta \geq 2$, and where one of the channels on link $(i + 1) \bmod N$ is channel $\{(c + 1) \bmod W$ and the other Δ channels on link $(i + 1) \bmod N$ are the channels $\{(c - k \cdot \Delta^i) \bmod W : k = 0, 1, \dots, \Delta - 1\}$.

9. A method as recited in claim 8 further comprising, a method of assigning channels to a set of end-to-end communication connections $1, \dots, j, \dots, m$, where each end-to-end communication connection j is a sequence of connected channels that follow a path p_j on the said ring communications network as recited in claim 8, a method of assigning channels to the paths $\{p_1, \dots, p_m\}$ further comprising:

- (a) identifying each path p_j as a cut path if it passes through said primary node and as an uncut path if it does not pass through said primary node;
- (b) for each cut path p_i , forming two paths a_i and b_i , referred to residual paths, by splitting p_i into two at the primary node, such that the primary node becomes an end node for both residual paths, and then labeling the residual path that traverses link 0 by b_i and labeling the residual path that traverses link $N - 1$ by a_i ;
- (c) partitioning the set of uncut and residual paths into W subsets denoted by (P_0, \dots, P_{W-1}) , such that in each subset no two paths traverse a common link;
- (d) finding a permutation $\pi(\cdot)$, as defined in claim 5d such that if p_j is a cut path, $a_j \in P_i$ and $b_j \in P_k$ then $\pi(i) = k$;
- (e) for the permutation $\pi(\cdot)$, defining a pair of channel numbers i and j to be π -

- related if there is a sequence of channel numbers (r_0, r_1, \dots, r_k) such that $r_0 = i$, $r_k = j$, and for $n = 0, 1, \dots, k-1$, $r_{n+1} = \pi(r_n)$;
- (f) partitioning the channel numbers $\{0, \dots, W-1\}$ into nonempty subsets (C_0, \dots, C_{M-1}) , where M is the number of subsets, such that numbers within a subset are π -related, while numbers from different subsets are not;
- (g) finding the size of each subset C_i and denoting it by s_i ;
- (h) for each C_i , finding a function $q_i(\cdot)$ that is defined for the set of values $\{0, \dots, s_i-1\}$ and takes values from the set $\{0, \dots, s_i-1\}$ such that:
- i. there is one element j in C_i such that $q_i(j) = 0$, and
 - ii. for each element j in C_i , $q_i(\pi(j)) = (q_i(j) + 1) \bmod s_i$;
- (i) finding a set of numbers $\{t_0, \dots, t_{M-1}\}$ from the set $\{0, \dots, W-1\}$ such that for $i = 0, \dots, M-1$, $t_{(i+1) \bmod M} = (t_i + s_i) \bmod W$;
- (j) for each $k = 0, \dots, M-1$:
- i. letting $(d_{N-1}(k), d_{N-2}(k), \dots, d_0(k))$ denote the base Δ , N digit representation of the value $s_k - 1$,
 - ii. letting $D_0(k)$ denote 0, and for $i = 1, 2, \dots, N-1$, letting $D_i(k)$ denote $\sum_{n=0}^{i-1} d_n(k) \cdot \Delta^n$; and
- (k) for each pair (i, j) , where $j \in \{0, 1, \dots, N-1\}$ is a link number and $i \in \{0, \dots, W-1\}$, determining a value $\sigma(i, j)$ comprised:
- i. let k denote the value such that $i \in C_k$,
 - ii. determine a value $\rho(i, j)$, where

- A. if $q_k(i) = s_k - 1$ then let $\rho(i, j) = s_k - 1 - D_j(k)$
- B. if $q_k(i) < s_k - 1$ and $q_k(i) < s_k - 1 - D_j(k)$ then let $\rho(i, j) = q_k(i)$,
- C. if $q_k(i) < s_k - 1$ and $q_k(i) \geq s_k - 1 - D_j(k)$ then let $\rho(i, j) = q_k(i) + 1$,
- iii. let $\sigma(i, j) = (t_k + \rho(i, j)) \bmod W$;
- (l) for each uncut path p_k and each link j it traverses, assign channel $\sigma(i, j)$ from the link to the path, where i is such that $p_k \in P_i$; and
- (m) for each residual path a_k (resp., b_k) and each link j it traverses, assign channel $\sigma(i, j)$ from the link to path p_k , where i is such that $a_k \in P_i$ (resp., $b_k \in P_i$).
10. A ring communications network having N nodes with multichannel communication links between each pair of said nodes, where each link has W channels, said ring network configured as follows:
- each of said nodes configured such that channel C on link i may be connected to one of $\Delta + 1$ channels on link $(i + 1) \bmod N$, where $\Delta \geq 2$ and where one of the channels on link $(i + 1) \bmod N$ is channel $(c + 1) \bmod W$ and the other Δ channels on link $(i + 1) \bmod N$ are the channels $(c - k \cdot \Delta^i) \bmod W$, where $k = 0, 1, \dots, \Delta - 1$, where the nodes are numbered from $0, 1, \dots, N$ starting at the primary node of the ring and proceeding in one direction around said ring, and where a link between nodes i and $(i + 1) \bmod N$ is designated as link i .
11. In a star network having a plurality of N nodes interconnected by a plurality of links, with one of said nodes being the *hub* node h and the other said nodes being $\{x_1, \dots, X_{N-1}\}$, referred to as the *spoke nodes*, and being connected to said hub node

by one of said links having W channels, where W is even, a method of configuring said nodes, said method comprising:

- (a) dividing channels into two sets, with each set having $W/2$ channels, where the first set has channels numbered $\{0, \dots, W/2 - 1\}$ and the second set has channels numbered $\{W/2, \dots, W - 1\}$; and
- (b) configuring said hub node such that channel i on any one of said links may be connected to channel $w(i)$ on any of said links, where $w(i)$ equals $i + W/2$.

12. A method as recited in claim 11, further comprising a method for assigning channels to a connection which traverses at most two of said links, wherein paths p_1, \dots, p_m traverse exactly two of said links and paths p_{m+1}, \dots, p_M traverse only one of said links, and wherein said links are designated as links e_1, e_2, \dots, e_{N-1} such that for $i = 1, \dots, N - 1$, e_i is between nodes x_i and h , comprising:

- (a) referring to a path as being *incident* to its end nodes;
- (b) *directing* paths $\{p_1, \dots, p_m\}$ so that each path is viewed as going from one of its end nodes to the other and that each spoke node will have at most $W/2$ incident paths that are directed into it and at most $W/2$ incident paths that are directed out of it;
- (c) referring to a node that has at least one incident undirected path as a *free* node;
- (d) directing the said paths $\{p_1, \dots, p_m\}$ by using the following procedure:
 - i. if each link has exactly W paths from the set $\{p_1, \dots, p_M\}$ that traverse it then let $R = M$; otherwise, construct additional paths $p_{M+1}, p_{M+2}, \dots, p_R$ in

the star network so that for each link, there are exactly W paths from the set $\{p_1, \dots, p_R\}$ that traverse it,

- ii. initially all paths $\{p_1, \dots, p_R\}$ are considered *undirected*, and
- iii. as long as there is a free node, do the following:
 - A. start from a free node, say x_i , and traverse an undirected incident path (from the set $\{p_1, \dots, p_R\}$) to the other end node, and direct the path in the direction of the traversal,
 - B. starting from the other end node, traverse an undirected incident path (from the set $\{p_1, \dots, p_R\}$) to the next end node, and direct the path in the direction of the traversal, and
 - C. keep traversing undirected paths (and directing the traversed paths) in this way until x_i is reached;
- (e) creating a bipartite graph G which has two sets of vertices $\{u_1, \dots, u_{N-1}\}$ and $\{v_1, \dots, v_{N-1}\}$ and has edges $\{b_1, \dots, b_m\}$ such that for $i = 1, \dots, m$, b_i is between u_j and v_k if path p_i is directed so that it traverses link e_j and then e_k ;
- (f) assigning a number from said first set $\{0, \dots, W/2 - 1\}$ to said edges of graph G such that at any vertex in graph G has all of its incident edges assigned to distinct number of said first set, and denoting said number assigned to each edge b_i by $q(b_i)$; and
- (g) for $i = 1, \dots, m$, assigning channels to p_i where
 - i. the channels are $q(b_i)$ from link e_j and $w(q(b_i))$ from link e_k where j and k are such that u_j and v_k are the end vertices of b_i , where $w(i) = i + W/2$, and

(h) for $i = 1, 2, \dots, N - 1$, assigning distinct channels to all paths from the set $\{p_{m+1}, \dots, p_M\}$ that traverse the link e_i such that said channels are not already assigned to paths from $\{p_1, \dots, p_m\}$.

13. In a star network having N nodes with one of said nodes a hub node, wherein each of the other of said nodes is connected to said hub node by a multichannel link having W channels, where W is an even integer, a star network configured as follows:

said hub node configured such that channel i on any one of said links may be connected to channel $w(i)$ on any other of said links, where $w(i) = (i + W/2)$.

14. In a network consisting of N nodes and E links e_1, e_2, \dots, e_E , wherein each link between nodes is a multichannel multiplexed link, consisting of W channels $\{0, 1, \dots, W - 1\}$, where W is even, a method of configuring the nodes in the network, said method comprising:

- (a) grouping channels into two sets, $\{0, \dots, W/2 - 1\}$ and $\{W/2, \dots, W - 1\}$; and
- (b) at each node, for $i = 0, 1, \dots, W/2 - 1$, connecting channel i on one link to channel $w(i)$ on all the other links incident on that node, where $w(i) = i + W/2$.

15. A method as recited in claim 14, further comprising a method for assigning channels to connections $1, 2, \dots, m$ using paths p_1, \dots, p_m , wherein each of said paths traverses at most two of said links, where no two connections traversing the same one of said links are assigned to the same channel on said one link, said method comprising:

- (a) creating an equivalent star network with $E + 1$ nodes with the E nodes e'_1, e'_2, \dots, e'_E corresponding to the edges in the original network and the remaining node h being

the hub node; and

(b) creating an equivalent set of connections in the said star network p'_1, p'_2, \dots, p'_m

such that:

- i. if connection p_i uses link e_j in the original network then connection p'_i uses the link between nodes e'_j and h in the said star network,
- ii. if connection p_i uses links e_j and e_k in the original network then connection p'_i uses the following two links in the said star network: the link between nodes e'_j and h and the link between nodes e'_k and h ,
- iii. assigning channels to the p'_i according to the **method of claim 12**, and assigning the same set of channels to p_i as to p'_i , and
- iv. assigning channels to the set of paths p'_1, \dots, p'_m such that for $i = 1, 2, \dots, m$ if p'_i is assigned channel c on the link between nodes e'_j and h and is also assigned channel c' on the link between nodes h and e'_k in the said star network then c and c' are the channels assigned to path p_i for links e_j and e_k respectively.

16. A network having N nodes and E links for interconnecting said nodes, where each link is a multichannel multiplexed link having W channels, where W is even, a network configured as follows:

each node, for $i = 0, 1, \dots, W/2 - 1$, channel i on one incident link connected to channel $w(i)$ on all other incident links of said each node, where $w(i) = i + W/2$.

References

- [1] F. J. Janniello, R. A. Neuner, R. Ramaswami, and P. E. Green, "Multi-protocol optical fiber multiplexer for remote computer interconnection," in *OFC'95 Technical Digest*, 1995.
- [2] P. Raghavan and E. Upfal, "Efficient routing in all-optical networks," in *Proceedings of 26th ACM Symposium on Theory of Computing*, pp. 134–143, May 1994.
- [3] A. Birman and A. Kershenbaum, "Routing and wavelength assignment methods in single-hop all-optical networks with blocking," in *Proceedings of IEEE Infocom*, pp. 431–438, 1995.
- [4] R. Ramaswami and K. N. Sivarajan, "Routing and wavelength assignment in all-optical networks," *IEEE/ACM Transactions on Networking*, pp. 489–500, Oct. 1995. An earlier version appeared in *Proc. Infocom'94*.
- [5] I. Chlamtac, A. Ganz, and G. Karmi, "Lightpath communications: An approach to high-bandwidth optical WAN's," *IEEE Transactions on Communications*, vol. 40, pp. 1171–1182, July 1992.
- [6] M. Kovacevic and A. S. Acampora, "On the benefits of wavelength translation in all optical clear-channel networks," *IEEE JSAC/JLT Special Issue on Optical Networks*, vol. 14, pp. 868–880, June 1996.

- [7] A. Birman, "Computing approximate blocking probabilities for a class of optical networks," *IEEE JSAC/JLT Special Issue on Optical Networks*, vol. 14, pp. 852–857, June 1996.
- [8] N. Wauters and P. Demeester, "Design of the optical path layer in multiwavelength cross-connected networks," *IEEE JSAC/JLT Special Issue on Optical Networks*, vol. 14, pp. 881–892, June 1996.
- [9] K.-C. Lee and V. O. K. Li, "A wavelength-convertible optical network," *IEEE/OSA Journal on Lightwave Technology*, vol. 11, pp. 962–970, May/June 1993.
- [10] K.-C. Lee and V. O. K. Li, "Routing and switching in a wavelength convertible lightwave network," in *Proceedings of IEEE Infocom*, pp. 578–585, 1993.
- [11] C. Berge, *Graphs and Hypergraphs*. Amsterdam: North Holland, 1976.
- [12] V. E. Beneš, *Mathematical Theory of Connecting Networks*. New York: Academic Press, 1965.
- [13] T. Inukai, "An efficient SS/TDMA time slot assignment algorithm," *IEEE Transactions on Communications*, vol. 27, pp. 1449–1455, Oct. 1979.

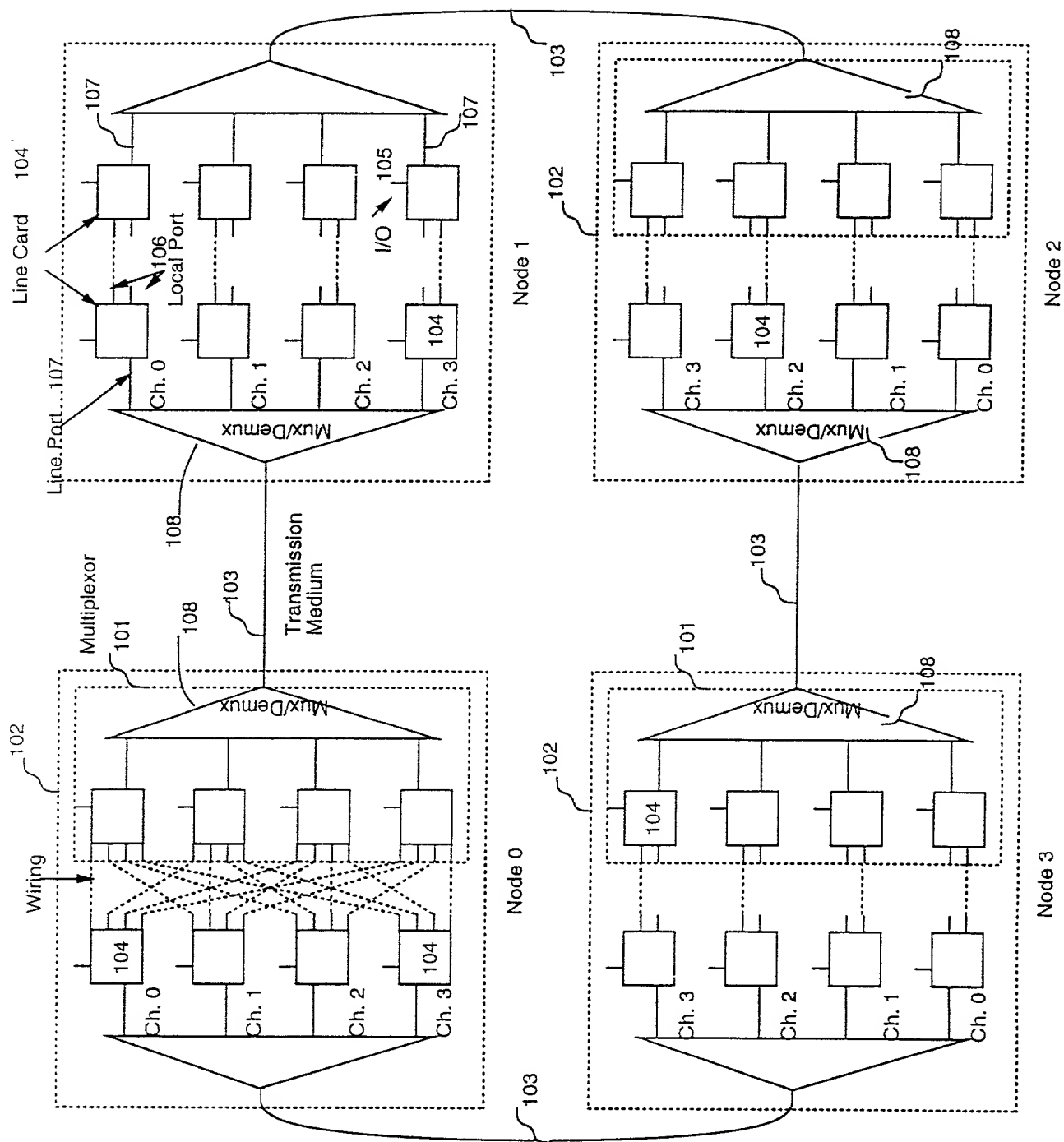


Figure 1:

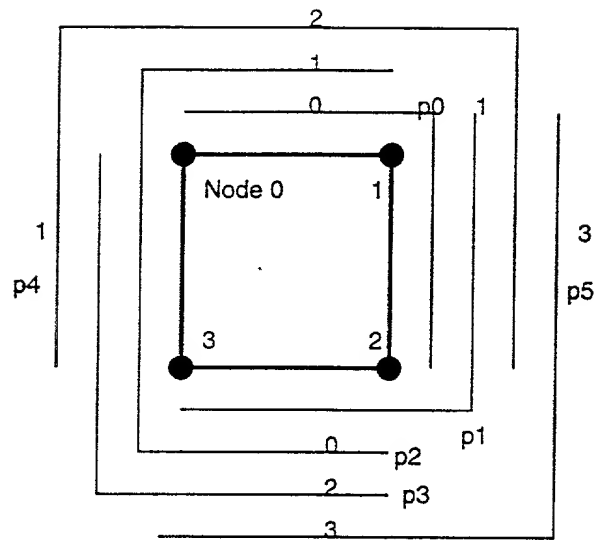


Figure 2:

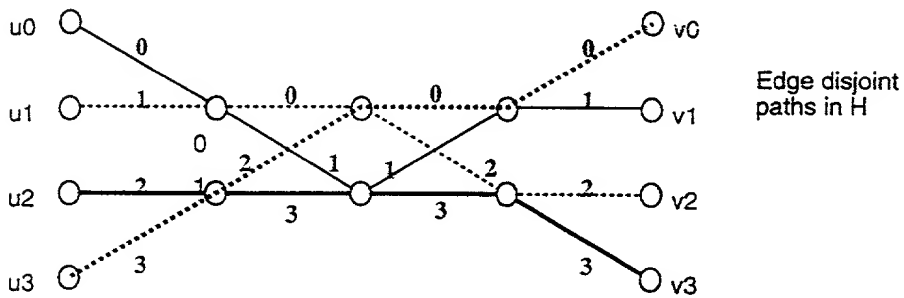
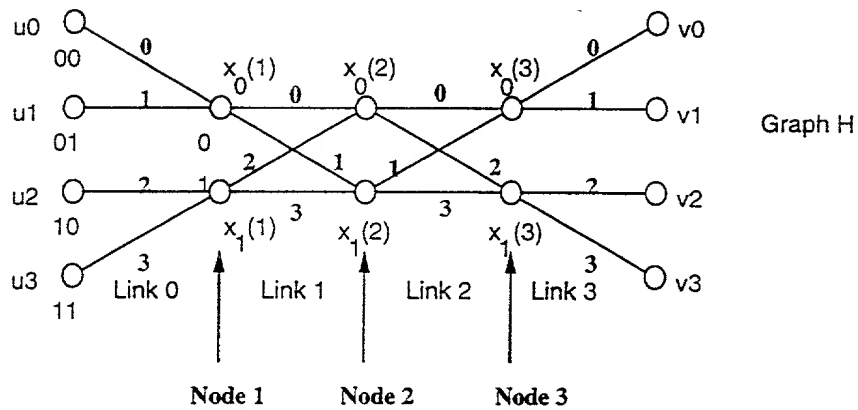


Figure 3:

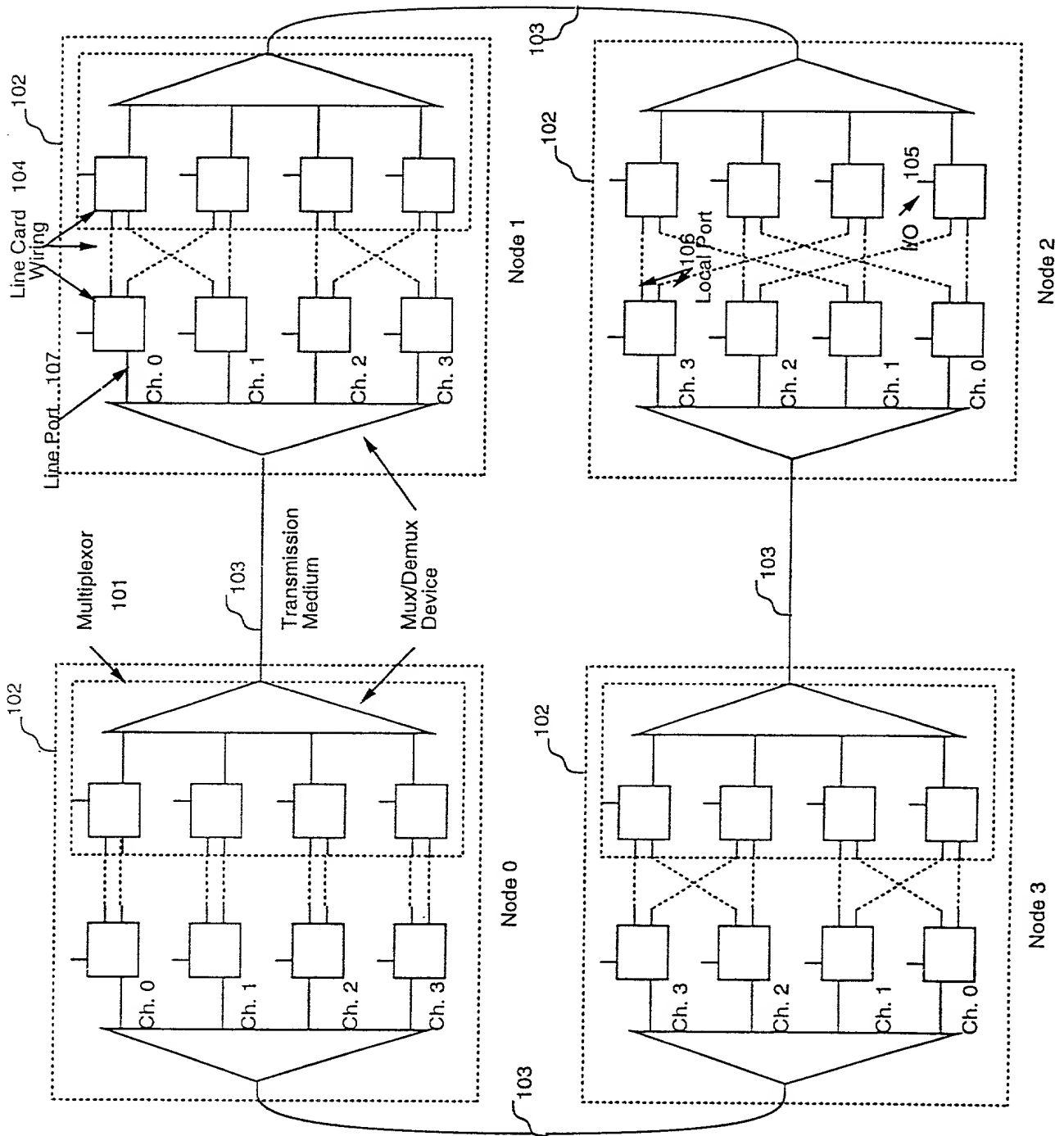


Figure 4:

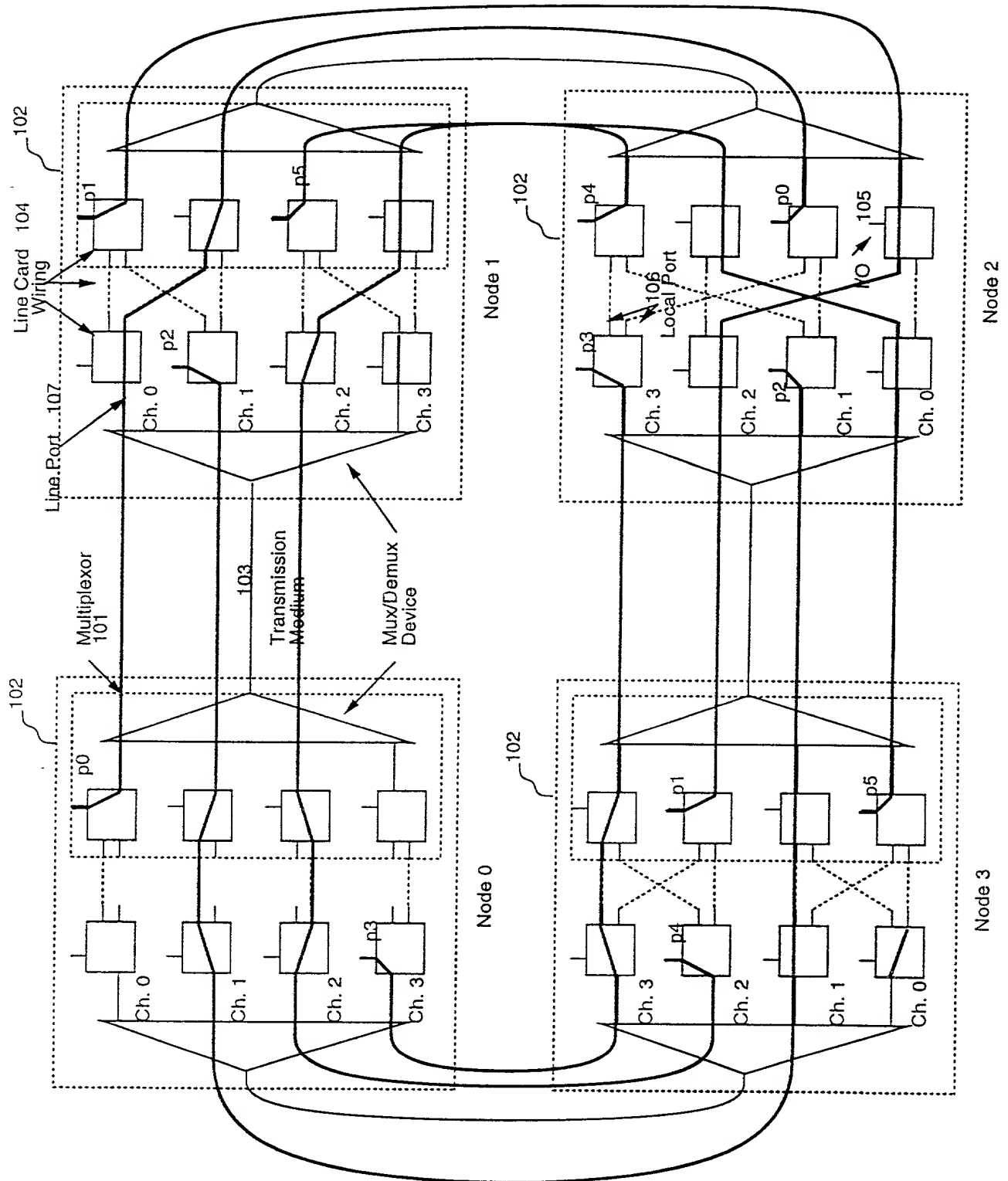


Figure 5:

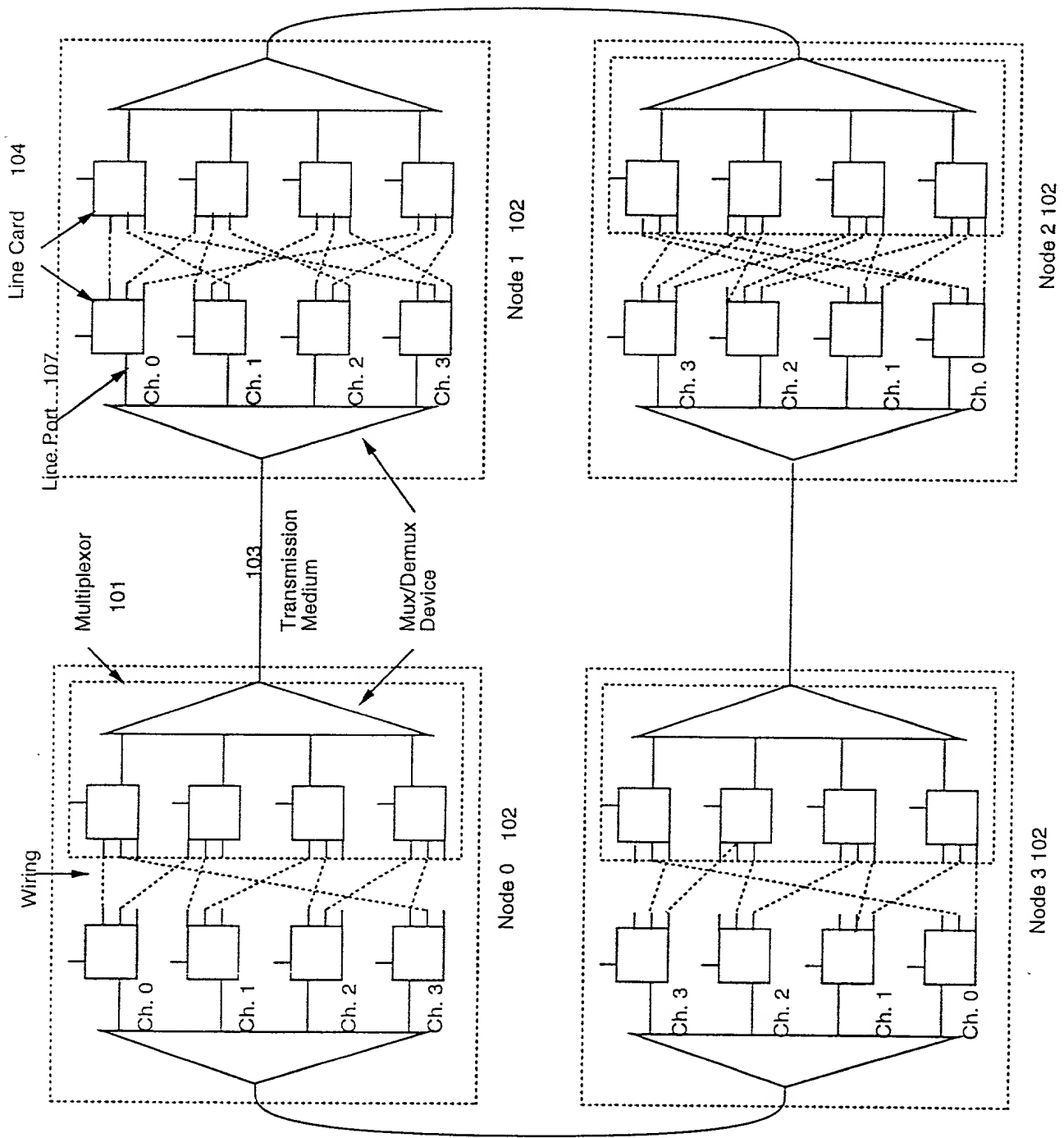


Figure 6:

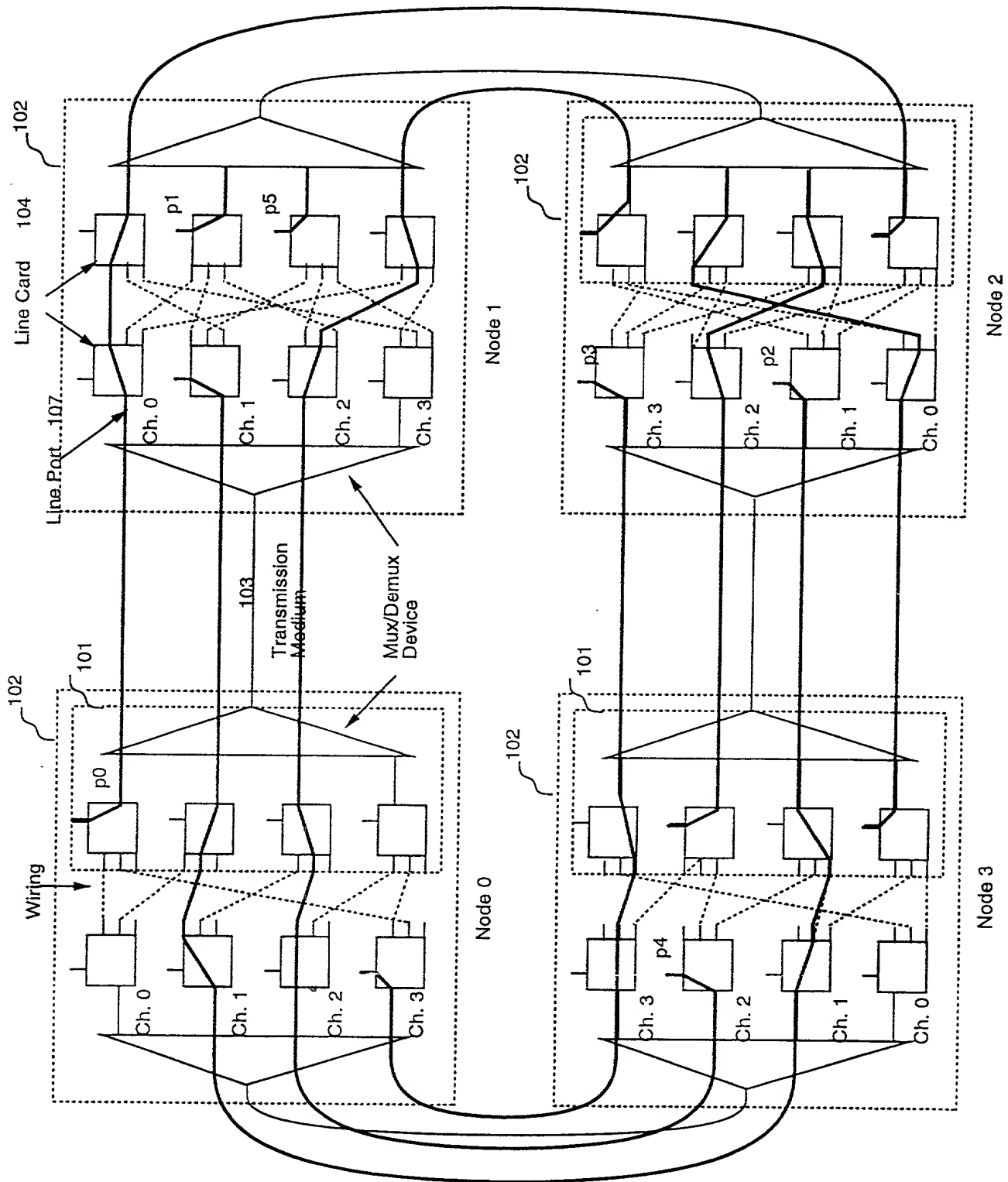


Figure 7:

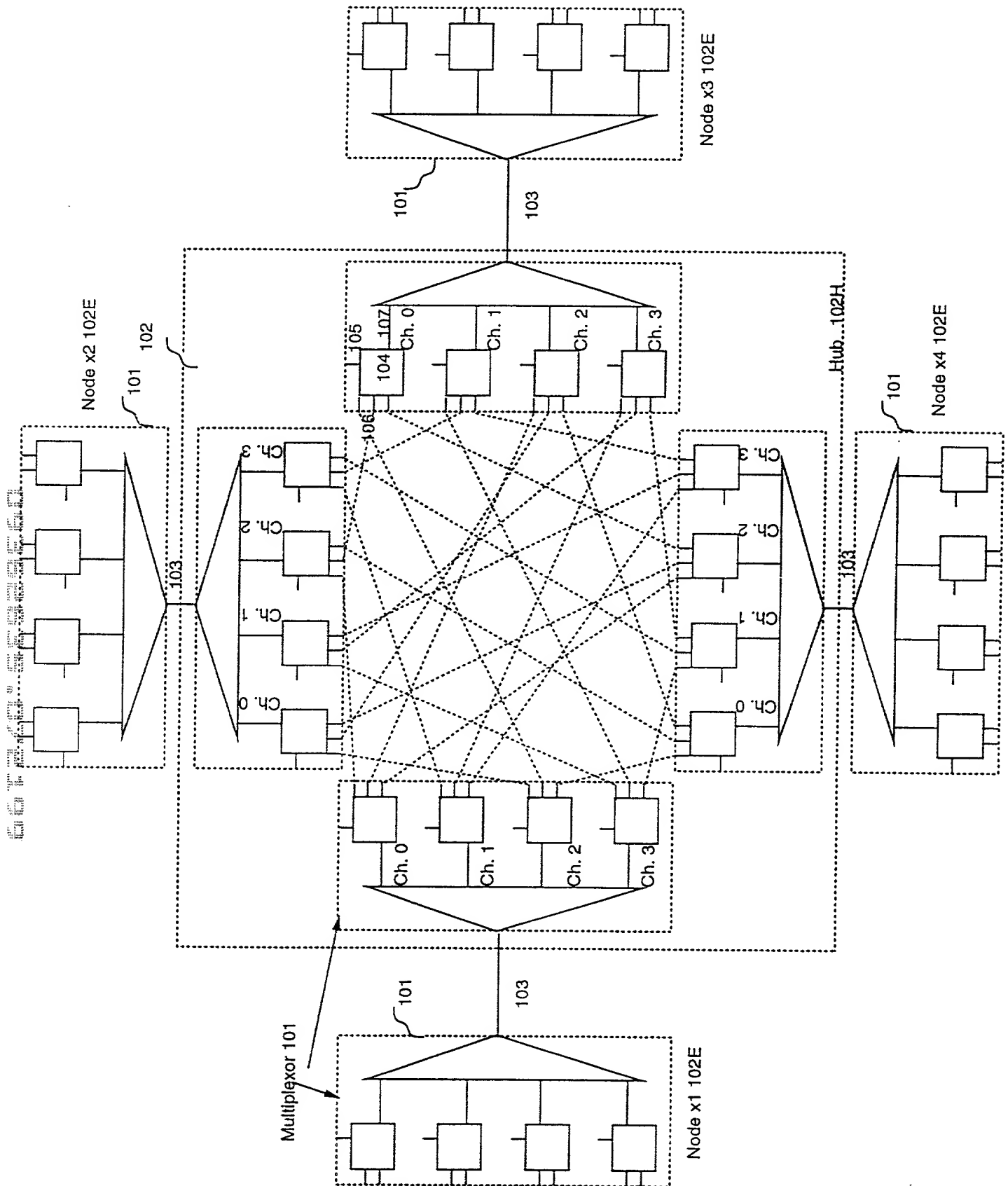


Figure 8:

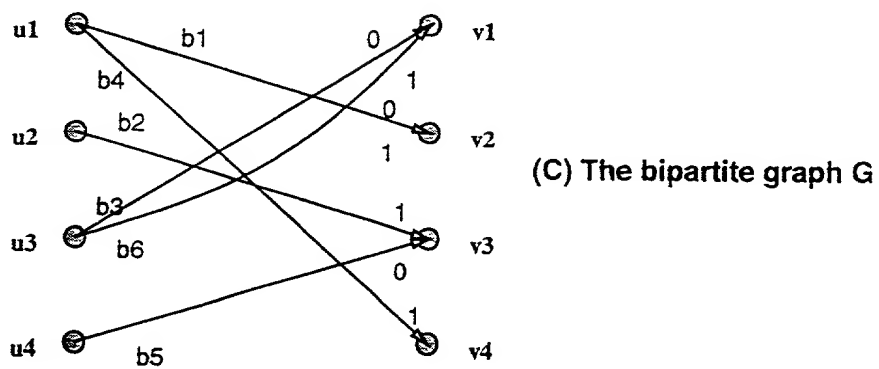
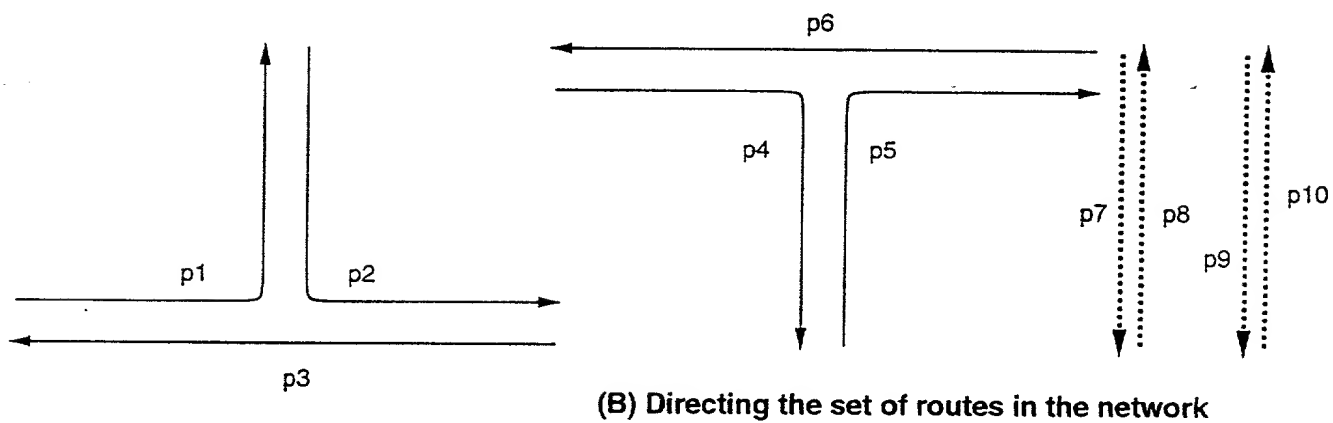
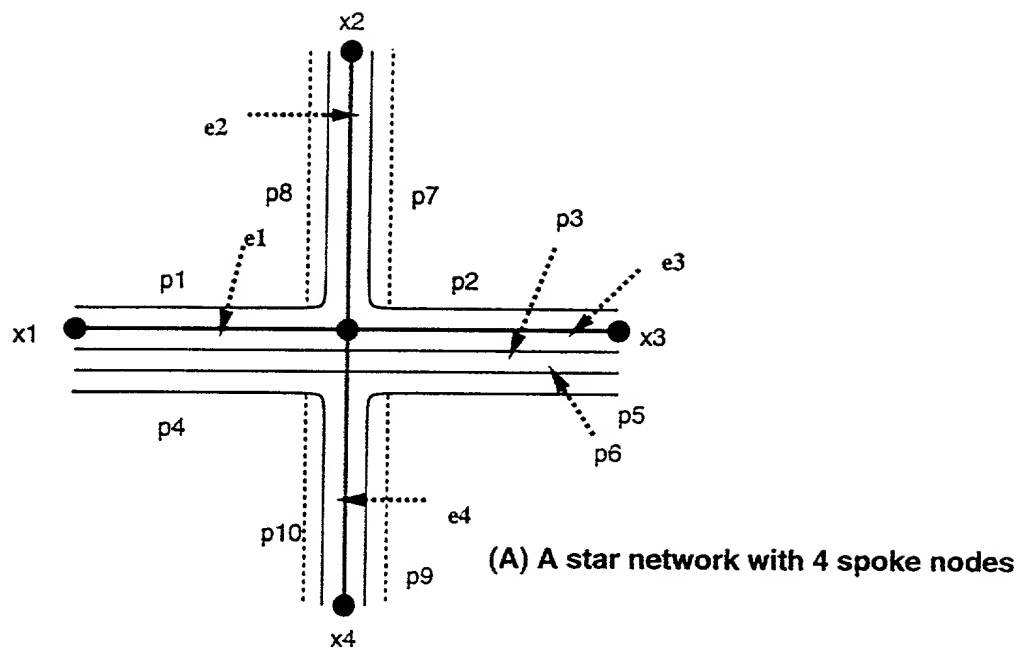


Figure 9:

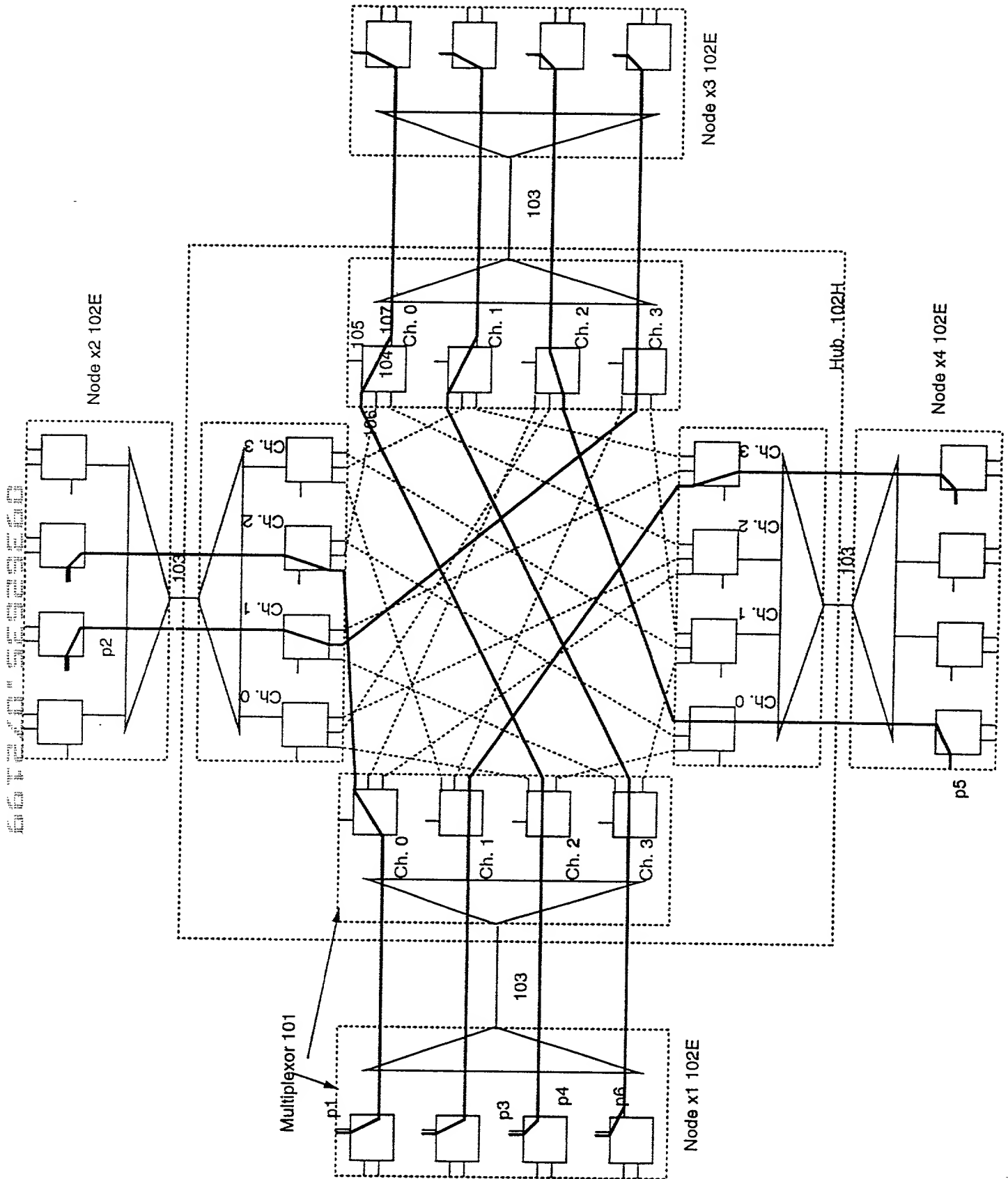


Figure 10:



DECLARATION AND POWER OF ATTORNEY FOR PATENT APPLICATION

As a below named inventor, I hereby are that:

My ~~trademark~~ post office address and citizenship are as stated below next to my name;

I believe I am the original, first and sole inventor (if only one name is listed below) or an original, first and joint inventor (if plural names are listed below) of the subject matter which is claimed and for which a patent is sought on the invention entitled:

MULTICHANNEL RING AND STAR NETWORKS WITH LIMITED CHANNEL CONVERSION
the specification of which (check one)

_____ is attached hereto.

XX was filed on April 29, 1996 as

Application Serial No. 08/641,061

and was amended on _____
(if applicable)

I hereby state that I have reviewed and understand the contents of the above identified specification, including the claims, as amended by any amendment referred to above.

I acknowledge the duty to disclose information which is material to the patentability of this application in accordance with Title 37, Code of Federal Regulations, Section 1.56.

I hereby claim foreign priority benefits under Title 35, United States Code, Section 119 of any foreign application(s) for patent or inventor's certificate listed below and have also identified below any foreign application for patent or inventor's certificate having a filing date before that of the application on which priority is claimed:

Prior Foreign Application(s)

Priority Claimed

(Number)	(Country)	(Day/Month/Year Filed)	<u>X</u> Yes	No
(Number)	(Country)	(Day/Month/Year Filed)	Yes	No
(Number)	(Country)	(Day/Month/Year Filed)	Yes	No

I hereby claim the benefit under Title 35, United States Code, Section 120 of any United States Application(s) listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of Title 35, United States Code, Section 112, I acknowledge the duty to disclose information material to the patentability of this application as defined in Title 37, Code of Federal Regulations, Section 1.56 which occurred between the filing date of the prior application and the national or PCT international filing date of this application:

(Application Serial No.)	(Filing Date)	(Status) (patented, pending, abandoned)
(Application Serial No.)	(Filing Date)	(Status) (patented, pending, abandoned)

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that willful false statements may jeopardize the validity of the application or any patent issued thereon.

POWER OF ATTORNEY: As a named inventor I hereby appoint the following attorney(s) and/or agent(s) to prosecute this application and transact all business in the Patent and Trademark Office connected therewith (list name and registration number).

3 Manny W. Schecter (Reg. No. 31,722) Marc D. Schechter (Reg. No. 28,989),
and Douglas W. Cameron (Reg. No. 31,596).

Send Correspondence to: Douglas W. Cameron Intellectual Property Law Dept.

IBM Corporation, P.O. Box 218, Yorktown Heights, New York 10598

Direct Telephone Calls to: (name and telephone number) Douglas W. Cameron (914) 945-3244

Rajiv Ramaswami 1-00
Full name of sole or first inventor

Inventor's Signature [Signature]

Date

52 Croton Avenue, Apt. 6C, Ossining, New York 10562 X64
Residence

India
Citizenship

Same as above.
Post Office Address

H. Galen Sasaki 2-00
Full name of second joint-inventor, if any

Galen H. Sasaki
Inventor's signature

Date

P. O. Box 10538, Honolulu, Hawaii 96816 H1
Residence

United States of America
Citizenship

Same as above.
Post Office Address

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

Case No. 11686US01

PATENT

In the Application of:

Rajiv Ramaswami et al.

Serial No.: 08/641,061

Filed: April 29, 1996

For: MULTICHANNEL RING AND
STAR NETWORKS WITH
LIMITED CHANNEL CONVERSION


Examiner:

Group Art Unit:

CERTIFICATE OF MAILING

I hereby certify that this correspondence and all correspondence referenced herein are being deposited with the United States Postal Service as first class mail in an envelope addressed to the Asst. Commissioner for Patents, United States Patent and Trademark Office, Washington DC 20231, on April 7, 1997.

By:


Robert B. Polit
Reg. No. 33,993

POWER OF ATTORNEY

Assistant Commissioner for Patents
Washington, D.C. 20231

Sir:

Tellabs, Inc., assignee of the above-captioned patent application, as manifest by Appendix H, hereby revokes all prior powers of attorney and appoints the following individuals as their representatives in connection with all matters relating to the application:

George P. McAndrews	Reg. No. 22,760
John J. Held, Jr.	Reg. No. 21,061
Timothy J. Malloy	Reg. No. 25,600
William M. Wesley	Reg. No. 26,521
Lawrence M. Jarvis	Reg. No. 27,341
Gregory J. Vogler	Reg. No. 31,313
Jean Dudek Kuelper	Reg. No. 30,171
Herbert D. Hart III	Reg. No. 30,063
Robert W. Fieseler	Reg. No. 31,826
D. David Hill	Reg. No. 35,543
Thomas J. Wimbiscus	Reg. No. 36,059
Steven J. Hampton	Reg. No. 33,707
Priscilla F. Gallagher	Reg. No. 32,223
Stephen F. Sherry	Reg. No. 30,590
Patrick J. Arnold Jr.	Reg. No. 37,769
Robert B. Polit	Reg. No. 33,993

George F. Wheeler Reg. No. 28,766
Janet M. McNicholas Reg. No. 32,918
Ronald E. Larson Reg. No. 24,478
Christopher C. Winslade Reg. No. 36,308 ✓
Edward A. Mas II Reg. No. 37,179
Gregory C. Schodde Reg. No. 36,668
Phyllis T. Brim Reg. No. 39,864
Sharon A. Hwang Reg. No. 39,717
Dean D. Small Reg. No. 34,730
Kirk A. Vander Leest Reg. No. 34,036
Anthony E. Dowell Reg. No. 39,661
Peter J. McAndrews Reg. No. 38,547
Daniel L. Pollmann Reg. No. 38,996.

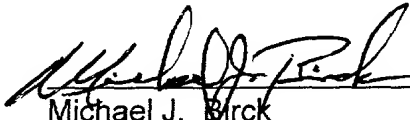
7

Please address all correspondence and direct all telephone calls to:

Robert B. Polit
McAndrews, Held & Malloy, Ltd.
500 W. Madison, 34th Floor
Chicago, IL 60661
312.707.8889.

Respectfully submitted,

4/3/97
Date


Michael J. Birck
President and CEO
Tellabs, Inc.

McAndrews, Held & Malloy, Ltd.
500 W. Madison Street, 34th Floor
Chicago, Illinois 60661
312.707.8889